NATIONAL ENGINEERING HANDBOOK

SECTION 4

HYDROLOGY

CHAPTER 17. FLOOD ROUTING

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1967

Revisions by

Wendell Styner

1972

Revised and reprinted, 1972

NEH Notice 4-102, August 1972

NATIONAL ENGINEERING HANDBOOK

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CHAPTER 17. FLOOD ROUTING

Contents Page

CONTENTS cont'd.

CONTENTS cont'd.

CONTENTS cont'd.

NATIONAL ENGINEERING HANDBOOK

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CHAPTER 17. FLOOD ROUTING

Introduction

In the American Society of Civil Engineers' manual, "Nomenclature for Hydraulics," flood routing is variously defined as follows:

routing (hydraulics).--(1) The derivation of an outflow hydrograph of a stream from known values of upstream inflow. The procedure utilizes wave velocity and the storage equation; sometimes both. (2) Computing the flood at a downstream point from the flood inflow at an upstream point, and taking channel storage into account.

routing, flood.-- The process of determining progressively the timing and shape of a flood wave at successive points along a river.

routing, streamflow.--The procedure used to derive a downstream hydrograph from an upstream hydrograph, or tributary hydrographs, and from considerations of local inflow by solving the storage equation.

Routing is also done with mass curves of runoff or with merely peak rates or peak stages of runoff, as well as hydrographs. The routing need not be only downstream because the process can be reversed for upstream routing, which is often done to determine upstream hydrographs from hydrographs gaged downstream. Nor is routing confined to streams and rivers; it is regularly used in obtaining inflow or outflow hydrographs, mass curves, or peak rates in reservoirs, farm ponds, tanks, swamps, and lakes. And low flows are routed, as well as floods. The term "flood routing" covers all of these practices.

The purpose of flood routing in most engineering work is to learn what stages or rates of flow occur, without actually measuring them, at specific locations in streams or structures during passages of floods. The stages or rates are used in evaluating or designing a water-control structure or project. Differences in stages or rates from routings made with and without the structure or project in place show its effects on the flood flows. In evaluations, the differences are translated into monetary terms to show benefits on an easily comparable basis; in design, the differences are used directly in developing or modifying the structure or project characteristics.

The routing process is based on one of the following approaches:

1. Solution of simultaneous partial differential equations of motion and continuity. Simplified versions of the equations are generally used in electronic computer routings; even the simplifications are too laborious for manual routings.
2. Solution of the continuity equation alone. A simplified form of the equation is the basis for many routing methods.
3. Use of inflow-outflow hydrograph relationships.
4. Use of unit hydrograph theory.
5. Use of empirical relationships between inflow and outflow peak stages or rates. Mostly used for large rivers.
6. Use of hydraulic models.

Methods based on the second, third, and fourth approaches are presented in this chapter. The routing operations in the methods can be made numerically by means of an electronic computer, desk calculator, slide rule, nomograph, network chart, or by mental calculations; or graphically by means of an analog machine, special chart, or by successive geometrical drawings. Methods specifically intended for electronic computers or analog machines are neither presented nor discussed.

All methods presented in this chapter are accurate enough for practical work if they are applied as they are meant to be and if data needed for their proper application are used. Advantages and disadvantages of particular methods are mentioned and situations that lead to greater or lesser accuracy of a method are pointed out, but there is no presentation of tests for accuracy or of comparisons between routed and gaged hydrographs.

SCS electronic computer program

The electronic computer program now being used in SCS watershed evaluations contains two methods of flood routing. The Storage-Indication method is used for routing through reservoirs and the Convex method for routing through stream channels. Manual versions of both methods are described in this chapter.

References

Each of the following references contains general material on flood routing and descriptions of two or more methods. References whose main subject is not flood routing but which contain a useful example of routing are cited in the chapter as necessary.

1. Thomas, H. A., 1937, The hydraulics of flood movements in rivers: Pittsburg, Carnegie Inst. Tech., Eng. Bull. Out of print but it can be found in most libraries having collections of engineering literature.

2. Gilcrest, B. R., 1950, Flood routing: Engineering Hydraulics (H. Rouse, ed.), New York, John Wiley and Sons, Chapter 10, pp. 635-710.

3. U.S. Department of the Army, Corps of Engineers, 1960, Routing of floods through river channels: Eng. Manual EM 1110-2-1408.

4. Carter, R.W., and R. G. Godfrey, 1960, Storage and flood routing: U.S. Geol. Survey Water-Supply Paper 1543-B.

5. Yevdjevich, Vujica M., 1964, Bibliography and discussion of flood-routing methods and unsteady flow in channels: U.S. Geol. Survey Water-Supply Paper 1690. Prepared in cooperation with the Soil Conservation Service.

6. Lawler, Edward A., 1964, Flood routing: Handbook of Applied Hydrology (V.T. Chow, ed.), New York, McGraw-Hill Book Co., section 25-II, pp. 34-59.

Summary of chapter contents

The remainder of this chapter is divided into four parts: elevation-storage and elevation-discharge relationships, reservoir routing methods, channel routing methods, and unit-hydrograph routing methods. In the first part, some relationships used in reservoir or channel routing are discussed and exhibits of typical results are given; in the second, the continuity equation is discussed and methods of using it in reservoir routings are shown in examples of typical applications; in the third, the theory of the Convex method is presented and examples of typical applications in channel routings are given; and in the fourth, the unit hydrograph theory is discussed and methods of applying it in systems analysis are shown in examples using systems of floodwater-retarding structures.

Elevation-Storage and Elevation-Discharge Relationships

In the examples of routing through reservoirs and stream channels it will be necessary to use elevation-storage or elevation-discharge curves (or both) in making a routing or as a preliminary to routing. Preparation of such curves is not emphasized in the examples because their construction is described in other SCS publications. The relationships are briefly discussed here as preliminary material; exhibits of tables and curves used in routings are given here and in some of the examples. Conversion equations used in preparing the tables and curves are given in Table 17-1.

Elevation storage relationships for reservoirs

Table 17-2 is a working table that shows data and computed results for an elevation-storage relationship to be used in some of the examples given later. Columns 1 and 7 or 1 and 8 give the relationship in different units of storage.

The relationship is developed from a contour map (or equivalent) of the reservoir area and the table is a record of the computations that were made. Once the map is available, the work goes as follows: (1) select contours close enough to define the topography with reasonable accuracy and tabulate the contour elevations in column 1; (2) determine the reservoir surface area at each elevation; for this table the areas were determined in square feet as shown in column 2 and converted to acres as shown in column 3; (3) compute average surface areas as shown in column 4; (4) tabulate the increments of depth in column 5; (5) compute the increments of storage for column 6 by multiplying an average area in column 4 by its appropriate depth increment in column 5; (6) accumulate the storage increments of column 6 to get accumulated storage in column 7 for each elevation of column 1; (7) convert storages of column 7 to storages in another unit, if required, and show them in the next column. The relationship of data in columns 1 and 8 is plotted in figure 17.1 as an elevation-storage curve.

Table 17-1. Equations for conversions of units

Conversion	Equation No.
cfs-hours = 12.1 (AF)	(Eq. 17.1)
cfs-days = 0.504 (AF)	(Eq. 17-2)
inches = (AF)/53.3 A	(Eq. 17-3)
$q_{id} = q_{cfs}/26.9\ A$	(Eq. 17-4)
$q_{ih} = q_{cfs}/645\ A$	(Eq. 17-5)
$q_{ad} = 1.98\ q_{cfs}$	(Eq. 17-6)
$q_{ah} = 0.0821\ q_{cfs}$	(Eq. 17-7)
$S_x = L\ (A_x)/3600$	(Eq. 17-8)
$S'_x = L\ (A_x)/297$	(Eq. 17-9)

where A = drainage area in square miles

A_x = cross section end-area in square feet for discharge x

AF = acre-feet

L = reach length in feet

q_{ad} = discharge in acre-feet per day

q_{ah} = discharge in acre-feet per hour

q_{cfs} = discharge in cfs

q_{id} = discharge in inches per day

q_{ih} = discharge in inches per hour

S_x = reach storage in cfs-hours for a given discharge x

S'_x = reach storage in acre-feet for a given discharge x

Table 17-2. Elevation-storage relationship for a reservoir.

Elevation (feet)	Surface area (sq.ft.)	Surface area (acres)	Average surface area (acres)	Δ depth (feet)	Δ storage (AF)	Storage (AF)	Storage (inches)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
570	0	0				0	0
			4.82	2.00	9.64		
572	420,000	9.64				9.64	.022
			18.36	2.00	36.72		
574	1,180,000	27.09				46.36	.109
			38.34	2.00	76.68		
576	2,374,000	54.50				123.04	.288
			71.62	4.00	286.49		
580	3,866,000	88.75				409.53	.960
			106.67	5.00	533.35		
585	5,427,000	124.59				942.88	2.210
			153.60	5.00	768.00		
590	7,954,000	182.60				1710.88	4.010
			205.64	5.00	1028.20		
595	9,961,000	228.67				2739.08	6.420
			250.01	5.00	1250.05		
600	11,820,000	271.35				3989.13	9.351

Elevation-discharge relationships for reservoirs

The elevation-discharge relationship for a reservoir is made using elevations of the reservoir and discharges of the spillways to be used in a routing. A typical relationship for a 2-stage principal spillway is given by columns 1 and 6 of Table 17-3 for discharges in cfs, and in columns 1 and 7 for discharges in in./day. The procedure for developing the relationship will not be given here because sufficient charts, equations, and examples for principal spillways are given in NEH-5 and in ES-150 through 153, and for emergency spillways in ES-98 and ES-124. Table 17-3 illustrates a useful way of keeping the work in order: by tabulating the data for different types of flow in separate columns, and by keeping the two stages separate, the total discharges are more easily summed. Note that the totals in cfs are not merely sums of all cfs in a row; the operation of the spillway must be understood when selecting the discharges to be included in the sum. To combine the principal spillway flow with emergency spillway flow a column for the emergency spillway discharges is added between columns 5 and 6, and totals in column 6 must include those discharges where appropriate. Column 7 gives discharges converted from those in column 6; it is shown because this table is used in examples given later and that particular unit of flow is required (see Figure 17-1).

Storage-discharge relationships for reservoirs

If the elevation-storage and elevation-discharge relationships are to be used for many routings it is more convenient to use them as a storage-discharge relationship. The relationships are combined by plotting a graph of storage and elevation, another of discharge and elevation, and, while referring to the first two graphs, making a third by plotting storage for a selected elevation against discharge for that elevation; for a typical curve see Figure 17-2. The storage-discharge curve can also be modified for ease of operations with a particular routing method; for a typical modification see Figure 17-6 and step 4 of Example 17-4.

Elevation, stage, storage, discharge relationships for streams

It is common practice to divide a stream channel into reaches (see Chapter 6) and to develop storage or discharge relationships for individual reaches rather than the stream as a whole. A stream elevation- or stage-discharge curve is for a particular cross section. If a reach has several cross sections within it they are all used in developing the working tools for routing. Some routing methods require the use of separate discharge curves for the head and foot of a reach; such methods are not presented in this chapter.

Elevation- or stage-discharge curves for cross sections or reaches are prepared as shown in Chapter 14. They will not be discussed here.

Elevation- or stage-storage curves for a reach can be prepared using the procedure for reservoirs but ordinarily a modified approach is used and the storage-discharge curve prepared directly. Table 17-4 is a working table for developing such a curve. The work is based on the assumption that steady flow occurs in the reach at all stages of flow. The reach used in Table 17-4 has four cross sections so that a weighting method

Table 17-3 Elevation-discharge relationship for a 2-stage principal spillway.

Elevation (feet)	First stage: Weir (cfs)	First stage: Orifice (cfs)	Second stage: Weir (cfs)	Second stage: Pipe (cfs)	Total (cfs)	Total (in./day)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
580.2	0				0	0
580.7	4.1				4.1	.019
581.2	11.6				11.6	.054
581.7	21.3				21.3	.099
582.2	32.8				32.8	.153
582.7	45.8				45.8	.213
583.2	60.3	0			60.3	.281
583.7	75.3	89.5			75.3	.350
584.2	92.8	101			92.8	.432
585.2	130	120			120	.559
586.0	162	133			133	.620
587.0	206	149	0	0	149	.694
587.5		159	44.6	343	204	.950
588.0		163	126	347	289	1.346
588.5		170	232	353	353	1.644
589.0		176	357	357	357	1.663
589.5		182	499	361	361	1.680
590.0			656	365	365	1.697
590.2			722	367	367	1.707
591.0				374	374	1.740
592.0				382	382	1.778
595.0				401	401	1.863
600.0				432	432	2.003

is needed; with only one or two sections the weighting is eliminated but the reach storage is less well defined. Development of the storage-discharge curve goes as follows: (1) select a series of discharges from zero to a discharge greater than any to be routed and tabulate them in column 1; (2) enter the stage-discharge curve for each cross section with a discharge from column 1 and find the stage; (3) enter the stage-end-area curve for that section with the stage from step 2 and find the area at that stage, tabulating areas for all sections as shown in columns 2, 3, 4, and 5; (4) determine the distances between cross sections and compute the weights as follows:

From cross section	To cross section	Distance (feet)	Weight
1	2	1000	0.10
2	3	6000	.60
3	4	3000	.30
	Sum:	10000	

with the weight for sub-reach 1-2 being 1000/10000 = 0.10, and so on; (5) compute weighted end areas for columns 6, 7, and 8; for example, at a discharge of 3,500 cfs cross section 1 has an end area of 2,500 square feet and section 2 has 640 square feet, and the weighted end area is 0.10(2500 + 640)/2 = 157 square feet; (6) sum the weighted areas of columns 6, 7, and 8 for each discharge, tabulating the sums in column 9; (7) compute storages in column 10 by use of Equation 17-8 or 17-9, whichever is required; for example, at a discharge of 3,500 cfs the storage in cfs-hrs is S_{3500} = 10000(1189)/3600 = 3300 cfs-hrs, by a slide-rule computation. The storage-discharge curve is plotted using data from columns 1 and 10. Data of those columns can be used in preparing the working curve for routing. How this is done depends on the routing method to be used. For the Storage-Indication method the working curve is prepared as shown in Example 17-4.

Table 17-4 Working table for a storage-discharge relationship

Out-flow	Cross section end-areas				Weighted end-areas			Avg. end-areas	Storage
	1	2	3	4	1-2	2-3	3-4		
(cfs)	(sq.ft)	(sq.ft)	(sq.ft)	(sq.ft)	(sq.ft)	(sq.ft)	(sq.ft)	(sq.ft)	(cfs-hrs)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	0	0	0	0	0	0	0	0	0
50	40	27	21	33	3	14	8	25	70
150	90	44	44	64	7	26	16	49	164
300	150	83	83	100	12	50	27	89	248
800	470	180	220	325	32	120	82	234	651
1500	950	310	460	700	63	231	174	468	1302
3500	2500	640	1200	2000	157	552	480	1189	3300
5000	3250	860	1700	2700	205	768	660	1633	4540
7000	4400	1050	2050	3400	272	930	819	2021	5620
10000	5800	1300	2550	4500	355	1155	1055	2565	7130

Reservoir Routing Methods

Reservoirs have the characteristic that their storage is closely related to their outflow rate. In reservoir routing methods the storage-discharge relation is used for repeatedly solving the continuity equation, each solution being a step in delineating the outflow hydrograph. A reservoir method is suited for channel routings if the channel has the reservoir characteristic. Suitable channels are those with swamps or other flat areas in the routing reach and with a constriction or similar control at the foot of the reach. There is an exception to this: a reservoir method is also suitable for routing through any stream reach if the inflow hydrograph rises and falls so slowly that nearly steady flow occurs and makes storage in the reach closely related to the outflow rate. Examples in this part show the use of reservoir methods for both reservoirs and stream channels.

The Continuity Equation

The continuity equation used in reservoir routing methods is concerned with conservation of mass: For a given time interval, the volume of inflow minus the volume of outflow equals the change in volume of storage. The equation is often written in the simple form:

$$\Delta t\ (\bar{I} - \bar{O}) = \Delta S \qquad \text{(Eq. 17-10)}$$

where Δt = a time interval

$\bar{I}$ = average rate of inflow during the time interval

$\bar{O}$ = average rate of outflow during the time interval

ΔS = change in volume of storage during the time interval

In most applications of the continuity equation the flow and storage variables are expanded as follows:

$$\bar{I} = \frac{I_1 + I_2}{2}\ ; \qquad \bar{O} = \frac{O_1 + O_2}{2}\ ; \qquad \Delta S = S_2 - S_1$$

so that Equation 17-10 becomes:

$$\frac{\Delta t}{2}(I_1 + I_2) - \frac{\Delta t}{2}(O_1 + O_2) = S_2 - S_1 \qquad \text{(Eq. 17-11)}$$

where $\Delta t = t_2 - t_1$ = time interval; t_1 is the time at the beginning of the interval and t_2 the time at the end of the interval

I_1 = inflow rate at t_1

I_2 = inflow rate at t_2

O_1 = outflow rate at t_1

O_2 = outflow rate at t_2

S_1 = storage volume at t_1

S_2 = storage volume at t_2

When routing with Equation 17-10 the usual objective is to find $\overline{O}$, with Equation 17-11 find O_2; this means that the equations must be rearranged in some more convenient working form. It is also necessary to use the relationship of outflow to storage in making a solution. Most reservoir routing methods now in use differ only in their arrangement of the routing equation and in their form of the storage-outflow relationship.

It is necessary to use consistent units with any routing equation. Some commonly used sets of units are:

Time	Rates		Volumes		
	Inflow	Outflow	Inflow	Outflow	Storage
Hours	cfs	cfs	cfs-hrs	cfs-hrs	cfs-hrs
days	cfs	cfs	cfs-days	cfs-days	cfs-days
days	AF/day	AF/day	AF	AF	AF
hours	in./hr	in./hr	inches	inches	inches
days	in./day	in./day	inches	inches	inches

Methods and Examples

Two methods of reservoir routing based on the continuity equation are presented in this section, a mass-curve method and the Storage-Indication method. The mass-curve method is given because it is one of the most versatile of all reservoir methods. It can be applied numerically or graphically; examples of both versions are given. The Storage-Indication method is given because it is the method used at the present time in the SCS electronic computer program for watershed evaluations and because it is a widely used method for both reservoir and channel routings. Examples of reservoir and channel routing are given.

Mass-Curve Method: Numerical Version - According to item 52 in reference 5, a mass-curve method of routing through reservoirs was already in use in 1883. Many other mass-curve methods have since been developed. The method described here is similar to a method given in King's "Handbook of Hydraulics," 3rd edition, 1939, pages 522-527; another resembling it is given in "Design of Small Dams," U. S. Bureau of Reclamation, 1960, pages 250-252.

The method requires the use of elevation-storage and elevation-discharge relationships either separately or in combination. The input is the mass (or accumulated) inflow; the output is the mass outflow, outflow hydrograph, and reservoir storage. The routing operation is a trial-and-error process when performed numerically, but it is simple and easily done. Each operation is a solution of Equation 17-10 rewritten in the form:

$$MI_2 - (MO_1 + \overline{O}\,\Delta t) = S_2 \qquad \text{(Eq. 17-12)}$$

where MI_2 = mass inflow at time 2

MO_1 = mass outflow at time 1

$\overline{O}$ = average discharge during the routing interval

Δt = routing interval = time 2 minus time 1

S_2 = storage at time 2

The routing interval can be either variable or constant. Usually it is more convenient to use a variable interval, making it small for a large change in mass inflow and large for a small change. The PSMC of Chapter 21 are tabulated in intervals especially suited for this method of routing.

The following example shows the application of the method in determining minimum required storage for a floodwater-retarding structure by use of a PSMC from Chapter 21.

Example 17-1.--Determine the minimum required storage, by SCS criteria, for a floodwater-retarding structure having the drainage area use in Example 21-2 of Chapter 21. Use the data and results of that example for this structure. Work with volumes in inches and rates in inches per day; round off all results to the nearest 0.01 inch.

1. Develop an elevation-discharge curve for the structure. A curve for the principal spillway discharges is needed for this routing. Columns 1 and 7 of Table 17-3 will be used for this structure. The elevation-discharge curve is plotted in Figure 17-1.

2. Develop an elevation-storage curve for the structure. Columns 1 and 8 of Table 17-2 will be used for this structure. The elevation-storage curve is plotted in Figure 17-1.

(Note: The curves of steps 1 and 2 can be combined into a storage-discharge curve as shown by the inset of Figure 17-2. This curve is a time-saver if more than one routing is made.)

3. Develop and plot the curve of mass inflow (PSMC). The PSMC developed in Example 21-2, and given by columns 1 and 7 of Table 21-7, will be used for this example. The plotted mass inflow is shown in Figure 17-2. The plotting is used as a guide in the routing and later used to show the results but it is not essential to the method.

4. Prepare an operations table for the routing. Suitable headings and arrangement for an operations table are shown in Table 17-5.

<u>5. Determine the reservoir storage for the start of the routing.</u>
If the routing is to begin with some storage already occupied then either the amount in storage is entered in the first line or column 5 of the operations table (as done in Example 17-2) or the elevation-storage curve is modified to give a zero storage for the first line. In this example the sediment or dead storage, which is not to be used in the routing, occupies the reservoir to elevation 580.2 feet as shown in Figure 17-1. Storage at that elevation is 1.00 inches and because this is a whole scale unit the storage curve for routing is easily obtained by shifting the point of origin as shown in Figure 17-1. Ordinarily, if the Sediment or dead storage is some fractional quantity it is better to re-plot the curve to show zero storage at the elevation where the routing begins.

<u>6. Determine the spillway discharge at the start of the routing.</u>
If the spillway is flowing at the start of the routing the discharge rate is entered in the first line of column 7 of Table 17-5 (see Example 17-2). For this example the starting rate is zero.

<u>7. Do the routing.</u>
The trial-and-error procedure goes as follows:

<u>a</u>. Select a time and tabulate it in column 1, Table 17-5. For this example the times used will be those given for the PSMC in Table 21-7, except for occasional omissions unimportant for this routing.

<u>b</u>. Compute Δt and enter the result in column 2.

<u>c</u>. Tabulate in column 3 the mass inflow for the time in column 1. The entries for this example come from column 7 of Table 21-7.

<u>d</u>. Assume a mass outflow amount and enter it in column 4.

<u>e</u>. Compute the reservoir storage, which is the inflow of column 3 minus the outflow of column 4, and enter it in column 5.

<u>f</u>. Determine the instantaneous discharge rate of the spillway. Using the elevation-storage curve of Figure 17-1, find the elevation for the storage of column 5; with that elevation enter the elevation-discharge curve and find the discharge, tabulating it in column 6. If a storage-discharge curve is being used, simply enter the curve with the storage and find the corresponding discharge.

<u>g</u>. Compute the average discharge for Δt. The average is always the arithmetic mean of the rate determined in step <u>f</u> and the rate for the previous time. For the time <u>0.5 days</u> the rate in column 6 is 0.03 in./day; for the previous time the rate is zero; the average rate is $(0 + 0.03)/2 = 0.015$, which

Table 17-5. Operations table for the mass-curve method of routing for Example 17-1.

Time Acc. (days)	Time Δt (days)	Acc. inflow (in.)	Assumed acc. outflow (in.)	Res. volume (in.)	Spillway discharge Inst. (in./day)	Spillway discharge Avg. (in./day)	Outflow for Δt (in.)	Acc. outflow (in.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0		0	0	0	0		0	0
.5	0.5	.13	.01	.12	.03	0.02	.01	.01
1.0	.5	.30	.04	.26	.08	.06	.03	.04
2.0	1.0	.69	.15	.54	.21	.14	.14	.18
			.17	.52	.19	.14	.14	.18
3.0	1.0	1.14	.40	.74	.31	.25	.25	.43
			.42	.72	.29	.24	.24	.42
3.5	.5	1.42	.60	.82	.36	.32	.16	.58
			.59	.83	.37	.33	.16	.58
4.0	.5	1.76	.75	1.01	.46	.42	.21	.79
			.78	.98	.44	.40	.20	.78
4.4	.4	2.11	1.02	1.09	.52	.48	.19	.97
			.98	1.13	.53	.48	.19	.97
4.8	.4	2.62	1.20	1.42	.61	.57	.23	1.20
5.0	.2	3.38	1.35	2.03	1.00	.80	.16	1.36
5.1	.1	4.07	1.45	2.62	1.66	1.33	.13	1.49
			1.48	2.59	1.65	1.32	.13	1.49
5.2	.1	4.43	1.70	2.73	1.68	1.66	.17	1.66
			1.67	2.76	1.68	1.66	.17	1.66
5.3	.1	4.66	1.85	2.81	1.69	1.68	.17	1.83
			1.84	2.82	1.69	1.68	.17	1.83
5.4	.1	4.81	2.10	2.71	1.67	1.68	.17	2.00
			2.01	2.80	1.68	1.68	.17	2.00
5.6	.2	5.05	2.30	2.75	1.67	1.68	.34	2.34
			2.33	2.72	1.67	1.68	.34	2.34
6.0	.4	5.38	2.80	2.58	1.66	1.66	.66	3.00
			2.95	2.43	1.64	1.66	.66	3.00
			3.00	2.38	1.60	1.64	.66	3.00
6.5	.5	5.70	3.80	1.90	.80	1.20	.60	3.60
			3.70	2.00	.94	1.27	.64	3.64
			3.65	2.05	1.04	1.32	.66	3.66
7.0	.5	5.98	4.10	1.88	.70	.89	.44	4.10
8.0	1.0	6.43	4.80	1.63	.66	.68	.68	4.79
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

Mass outflow is plotted using entries in column 4 or column 9. The outflow hydrograph is plotted using column 6, which gives instantaneous rates at the accumulated times shown in column 1.

is rounded to 0.02 in./day. For the time 1.0 days the average is (0.03 + 0.08)/2 = 0.055, which is rounded to 0.06 in./day; and so on.

h. Compute the outflow for Δt. Multiply the Δt of column 2 by the average rate of column 7 and get the increment of outflow for column 8.

i. Add the outflow increment of column 8 to the total of column 9 for the previous time and tabulate the sum in column 9.

j. Compare the mass outflow of column 9 with the assumed mass outflow of column 4. If the two entries agree within the specified degree of accuracy (0.01 inch, in this routing) then this routing operation is complete and a new one is begun with step a. If the two entries do not agree well enough then assume another mass outflow for column 4 and repeat steps e through j.

8. Determine the minimum required storage.
Examine the entries in column 5 and find the largest entry, which is 2.82 inches at 5.3 days. This is the minimum required storage.

The routing gives the reservoir storages in column 5, outflow hydrograph in column 6, and mass outflow in column 9, for the times of column 1. Unless the results are to be used in a report or exhibit, the routing is usually carried only far enough past the time of maximum storage to ensure that no larger storage will occur. The mass inflow and outflow for this example are plotted in Figure 17-2, with outflow shown only to 8.0 days. If the mass outflow plotting is made during the routing the trend of the curve indicates the best assumption for the next step in column 4.

The next example shows how the routing proceeds when it must start with the reservoir containing live storage and the spillway discharging.

Example 17-2.--For the same reservoir used in Example 17-1, determine the elevation and amount of storage remaining in the reservoir after 10 days of drawdown from the minimum level allowed by SCS criteria. The base flow used in developing the PSMC (see Example 21-2) is assumed to continue at the same rate throughout the routing. Round all work to the nearest 0.01 inch.

1. Determine the storage volume in the reservoir and the spillway discharge for the start of the routing.
SCS criteria permit the drawdown routing to start with storage at the maximum elevation attained in the routing of the PSH or PSMC used in determining the minimum required storage, even though the structure may be designed to contain more than the minimum storage. For this example the starting storage of 2.82 inches is found in column 5, and the associated discharge of 1.69 in./day in column 6, of Table 17-5 in the line for 5.3 days.

2. Prepare an operations table for the routing.
Ordinarily the suitable headings and arrangement are those of Table 17-5, but if base flow, snowmelt, or upstream releases must be included (base flow in this routing) then one or more additional columns are needed. Table 17-6 shows headings and arrangement suitable for this example.

3. Do the routing.
The procedure of step 7, Example 17-1, is slightly modified for this routing. The first line of data in the operations table must contain the initial reservoir volume in column 4 and the initial spillway discharge in column 7. Accumulated base flow is added to the initial value of column 4 to give the "accumulated inflow" of that column. In all other respects the routing procedure is that of step 7, Example 17-1.

4. Determine the storage remaining after 10 days of drawdown.
The entry in column 6 at day 10 shows that the remaining storage is 0.20 inches, which is at elevation 581.1 feet.

The routing for this example has been carried to 14 days to show that when the inflow rate is steady, as it is in this case (0.045 in./day), then the outflow rate eventually also becomes steady at the same rate. The larger the steady rate of inflow the sooner the outflow becomes steady. Note that if the routing had been done with an accuracy to the nearest 0.001 inch, the outflow rate would be 0.045 in./day, the base flow rate.

The mass inflow, storage, and mass outflow curves for this example are shown in Figure 17-3. Note that the work is accurate to the nearest 0.01 inch, therefore the curves must follow the plotted points within that limit. Slight irregularities in the smooth curves are due to slope changes in the storage-discharge curve.

Mass-Curve Method: Direct Version.- It is easy enough to eliminate the trial-and-error process of the mass-curve method but the resulting "direct version" is much more laborious than the trial-and-error version. To get a direct version the working equation is obtained from Equation 17-12 as follows.

The average discharge $\overline{O}$ in Equation 17-12 is $(O_1 + O_2)/2$ so that the equation can be written:

$$MI_2 - MO_1 - \frac{\Delta t}{2}(O_1 + O_2) = S_2 \qquad \text{(Eq. 17-13)}$$

Because O_2 as well as S_2 is unknown it is necessary to make combinations of S and O to get direct solutions in the routing operation. At any time, mass outflow is equal to mass inflow minus storage, or:

$$MO_1 = MI_1 - S_1 \qquad \text{(Eq. 17-14)}$$

Table 17-6 Operations table for determining storage after 10 days of drawdown for Example 17-2.

Time Acc. (days)	Time Δt (days)	Acc. base flow* (in.)	Acc. in-flow (in.)	As-sumed acc. outflow (in.)	Res. vol-ume (in.)	Spillway discharge Inst. (in./day)	Spillway discharge Avg. (in./day)	Out-flow for Δt (in.)	Acc. out-flow (in.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0		0	2.82		2.82	1.69			0
.2	0.2	.01	2.83	0.34	2.49	1.66	1.67	0.33	.33
.4	.2	.02	2.84	.64	2.20	1.35	1.50	.30	.63
.6	.2	.03	2.85	.92	1.93	.87	1.11	.22	.85
				.86	1.99	.98	1.16	.23	.86
1.0	.4	.04	2.86	1.20	1.66	.66	.82	.33	1.19
1.5	.5	.07	2.89	1.50	1.39	.60	.63	.32	1.51
2.0	.5	.09	2.91	1.80	1.11	.53	.56	.28	1.79
2.5	.5	.11	2.93	2.03	.90	.37	.45	.22	2.01
				2.01	.92	.38	.46	.23	2.02
3.0	.5	.14	2.96	2.23	.73	.27	.32	.16	2.18
				2.19	.77	.29	.34	.17	2.19
3.5	.5	.16	2.98	3.30	.68	.24	.26	.13	2.32
				2.32	.66	.23	.26	.13	2.32
4.0	.5	.18	3.00	2.42	.58	.20	.22	.11	2.43
4.5	.5	.20	3.02	2.52	.50	.17	.18	.09	2.52
5.0	.5	.22	3.04	2.59	.45	.15	.16	.08	2.60
6.0	1.0	.27	3.09	2.73	.36	.12	.14	.14	2.74
7.0	1.0	.32	3.14	2.85	.29	.09	.10	.10	2.84
8.0	1.0	.36	3.18	2.94	.24	.07	.08	.08	2.92
				2.93	.25	.08	.08	.08	2.92
9.0	1.0	.40	3.22	3.00	.22	.07	.08	.08	3.00
10.0	1.0	.45	3.27	3.07	.20	.07	.07	.07	3.07
11.0	1.0	.50	3.32	3.13	.19	.06	.06	.06	3.13
12.0	1.0	.54	3.36	3.19	.17	.05	.06	.06	3.19
13.0	1.0	.58	3.40	3.25	.15	.04	.04	.04	3.23
				3.24	.16	.05	.05	.05	3.24
14.0	1.0	.63	3.45	3.29	.16	.05	.05	.05	3.29
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

* At a rate of 0.045 inches per day.

Substituting $MI_1 - S_1$ for MO_1 in Equation 17-13 and rearranging gives:

$$MI_2 - MI_1 + (S_1 - \frac{\Delta t}{2} O_1) = S_2 + \frac{\Delta t}{2} O_2 \qquad \text{(Eq. 17-15)}$$

which is the working equation for the direct version. Working curves of O_1 and $(S_1 - (\Delta t\ O_1)/2)$ and of O_2 and $(S_2 + (\Delta t\ O_2)/2)$ are needed for routing.

Other arrangements of working equations can also be obtained from Equation 17-12. Equation 17-15 is the mass-curve version of the Storage-Indication method, which is described later in this part. Routing by use of Equation 17-15 takes about twice as much work as routing by the Storage-Indication method.

Examples of direct versions of the mass-curve method are not given in this chapter because the trial-and-error version is more efficient in every respect.

Mass-Curve Method: Graphical Version.- The graphical version of the mass-curve method is in a sense a direct version because there is no trial-and-error involved. The graphical version is usually faster than the trial-and-error version if the routing job is simple. For complex jobs the trial-and-error version is more efficient and its results more easily reviewed. For any routing it gives mass outflow, storage, and the outflow hydrograph; the graphical version gives only the mass outflow and storage. The following example shows the use of the graphical version with the data and problem of Example 17-1.

Example 17-3.--Use the graphical version of the mass-curve method to determine the minimum required storage for the structure used in Example 17-1. Use the data of that example.

1. Develop an elevation-discharge curve for the structure.
The curve used in Example 17-1 will be used here.

2. Develop an elevation-storage curve for the structure.
The curve used in Example 17-1 will be used here.

3. Prepare a working table for the routing.
Using the curves of steps 1 and 2, select enough discharges on the discharge curve to define the curve accurately and tabulate them in column 2, Table 17-7. Tabulate the associated elevations in column 1 and storages at those elevations in column 4. Compute average discharges from column 2 for column 3. The designations in column 5 show which line is associated with each pair of storages shown on Figure 17-4. Thus, line A applies when the storage is between 0 and 0.18 inches; line B when it is between 0.18 and 0.40 inches; and so on.

4. Plot the mass inflow.
The PSMC used in Example 17-1 is used here. It is plotted in Figure 17-4.

5. Do the routing.
The work is done on the graph of mass inflow, Figure 17-4. Table 17-7 is used during the work. The procedure goes as follows:

a. Draw line A with its origin at the beginning of mass inflow and with its slope equal to the associated average discharge (column 3 of Table 17-7), which is 0.025 in./day. This is the first portion of the mass outflow curve.

(Note: Every part of the line of mass outflow must fall on or below the mass inflow curve. If some part is above the inflow, determine the slope and storage limits for a line with a flatter slope and use it instead.)

b. Determine the time at which the difference between mass inflow and line A is equal to the larger of the storage limits for line A, in this case 0.18 inches, which occurs at 0.65 days. This is the point of origin for line B.

c. Draw line B with its origin at the point found in step b and with a slope of 0.09 in./day.

d. Determine the time at which the difference between mass inflow and line B is equal to the larger of the storage limits for line B, in this case 0.40 inches, which occurs at 1.50 days. This is the point of origin for line C.

e. Repeat the procedure of steps c and d with lines C, D, E, etc., until the storage being used is so large it exceeds the possible difference between mass inflow and mass outflow. For this example this occurs with line H. The parallel line above it shows that the associated storage of 3.44 inches falls above the mass inflow line. When this step is reached the required storage is obtained by taking the maximum difference between line H and the mass inflow curve. The difference occurs at the point on the mass inflow curve where a line parallel to line H is tangent to the inflow curve. For this example it is 2.80 inches at 5.33 days. This step completes the routing.

The graphical method can also be used for routings starting with some storage occupied and with the spillway discharging. For the problem used in Example 17-2 the graphical method starts with line H and continues with lines G, F, E, D, C, B, and A in that order. The results are shown in Figure 17-5. The storage after 10 days of drawdown is 0.18 inches, which is nearly the same as found in Example 17-2. Differences between results of the two methods are due mainly to the use of small-scale graphs for working curves; larger scales increase the accuracy. Note that line A in Figure 17-5 is flatter than the line of accumulated base flow. This indicates that the flow becomes steady at or near 10 days and that the dashed line (parallel to mass inflow) is the actual outflow.

Table 17-7 Working table for the graphical version of the mass-curve method for Example 17-3.

Elevation	Spillway discharge Inst.	Spillway discharge Avg.	Storage	Designation on Fig. 17-4
(feet)	(in./day)	(in./day)	(inches)	
(1)	(2)	(3)	(4)	(5)
580.2	0		0	
		0.025		line A
581.0	.05		.18	
		.09		line B
582.0	.13		.40	
		.23		line C
583.5	.33		.80	
		.42		line D
584.6	.52		1.09	
		.61		line E
587.0	.70		1.86	
		.95		line F
587.8	1.20		2.16	
		1.42		line G
588.4	1.64		2.38	
		1.69		line H
591.0	1.74		3.44	
		1.77		line I
592.5	1.80		4.15	

Storage-Indication Method.- Reservoir routing methods that are also used for stream routings are generally discharge, not mass, methods because it is usually only the discharge hydrograph that is wanted. The Storage-Indication method, which has been widely used for channel and reservoir routings, has discharge rates as input and output. The method was given in the 1955 edition of NEH-4, Supplement A. Example 17-4, below is the same example used in that publication except for minor changes.

The Storage-Indication method uses Equation 17-11 in the form:

$$\bar{I} + \frac{S_1}{\Delta t} - \frac{O_1}{2} = \frac{S_2}{\Delta t} + \frac{O_2}{2} \qquad \text{(Eq. 17-16)}$$

where $\bar{I} = (I_1 + I_2)/2$. The values of $\bar{I}$ are either taken from midpoints of routing intervals of plotted inflow hydrographs or computed from inflows tabulated at regular intervals. A working curve of O_2 plotted against $(S_2/\Delta t) + (O_2/2)$ is necessary for solving the equation.

In channel routing the Storage-Indication method has the defect that outflow begins at the same time inflow begins so that presumably the inflow at the head of the reach passes instantaneously through the reach regardless of its length. This defect is not serious if the ratio T_t/T_p is about 1/2 or less, where T_p is the inflow hydrograph time to peak and T_t is a travel time defined as:

$$T_t = \frac{L\ A}{3600\ q} = \frac{L}{3600\ V} \qquad \text{(Eq. 17-17)}$$

where T_t = reach travel time in hours; the time it takes a selected steady-flow discharge to pass through the reach

L = reach length in feet

A = average end-area for discharge q in square feet

q = selected steady-flow discharge is cfs

V = q/A = average velocity of discharge q in fps

In determining T_t the discharge q is usually the bank-full discharge under steady flow conditions (see Chapter 15).

Another defect of the Storage-Indication method, for both channel and reservoir routing, is that there is no rule for selecting the proper size of routing interval. Trial routings show that negative outflows will occur during recession periods of outflow whenever Δt is greater than $2\ S_2/O_2$ (or whenever $O_2/2$ is greater than $S_2/\Delta t$). This also means that rising portions of hydrographs are being distorted. In practice, to avoid these possibilities, the working curve can be plotted as shown in Figure 17-6; if any part of the working curve falls above the line of equal values then the entire curve should be discarded and a new one made using a smaller value of Δt. For channel routing the possibility of negative outflows is usually excluded by taking Δt less than T_t.

The following example shows the use of the Storage-Indication method in channel routing. The example is the one used in the 1955 edition of NEH-4, Supplement A, with some minor changes.

Example 17-4.--Use the Storage-Indication method of reservoir routing to route the inflow hydrograph of Figure 17-7 through the stream reach of Table 17-4.

1. Prepare the storage-discharge relationship for the reach.
This is done in Table 17-4 and the text accompanying it.

2. Determine the reach travel time.
This is done using Equation 17-17. Table 17-4 and the accompanying text supply the following data: L = 10,000 feet and for a bank-full discharge of 800 cfs as q the end-area A = 234 square feet. Then by Equation 17-17, $T_t = 10000(234)/3600(800) = 0.813$ hours.

3. Select the routing interval.
The routing interval for this example will be 0.5 hours, which is less than the travel time of step 2 and which is a convenient size for the given inflow hydrograph. (See the discussion in the text accompanying Equation 17-30 for further information on the selection of reach routing intervals.)

4. Prepare the working curve.
Use the storage-discharge relationship of step 1, which is given in columns 1 and 10 of Table 17-4. These two columns are reproduced as columns 1 and 3 of Table 17-8, the working table; columns 2, 4, and 5 of the table are self-explanatory. The working curve is plotted using columns 1 and 5. The finished curve is shown in Figure 17-6.

5. Prepare the operations table.
Suitable headings and arrangement for an operations table are shown in Table 17-9.

6. Enter times and inflows in the operations table.
Accumulated time in steps of the routing interval is shown in column 1 of Table 17-9. $\bar{I}$ values read from midintervals on the inflow hydrograph of Figure 17-7 are shown in column 2.

7. Do the routing.
The procedure is shown in Table 17-10. The routing results are shown in columns 3 and 4 of Table 17-9. The outflow hydrograph given in column 4 is plotted in Figure 17-7.

In routing through channels it is generally necessary to add local inflow to the routed outflow. The method of doing this is described later in the part on channel routing methods.

The Storage-Indication procedure for reservoir routing is identical with that for channel routing except that there is no need to determine a travel time. The following example shows the reservoir procedure. The problem and data of Example 17-1 are used in order to allow a comparison of procedures and results.

Example 17-5.--Use the Storage-Indication method to determine the minimum required storage for the structure used in Example 17-1. Use the data of that example where applicable. Make the routing with discharges in cfs.

1. Develop an elevation-discharge curve for the structure.
The curve used in Example 17-1 will be used here. That curve is for discharges in in./hr. Ordinarily when cfs are to be used the curve is developed in that unit. The conversion to cfs will be made in step 5.

2. Develop an elevation-storage curve for the structure.
The curve used in Example 17-1 will be used here. That curve is for storage in inches. The conversion to cfs-days will be made in step 5.

3. Develop and plot the inflow hydrograph.
Because of the type of problem the inflow hydrograph must be a Principal Spillway Hydrograph (PSH) taken from Chapter 21. The PSH corresponding to the PSMC of Example 17-1 is given in columns 1 and 4 of Table 21-7. The PSH is plotted in Figure 17-8.

4. Select the routing interval.
Examination of the PSH in Figure 17-8 shows that two routing intervals will be needed, one of 0.5 days for small changes in rates and one of 0.1 days for large changes.

5. Prepare the working curves.
Data and computations for the working curves are shown in Table 17-11. Two curves are needed because two routing intervals will be used. The elevations of column 1 and discharges of column 2 are taken from the curve of step 1 with the discharges being converted from in./hr. to cfs in the process. The discharges are selected so that they adequately define the elevation-discharge relationship. Column 3 of Table 17-11 gives the corresponding storages from the curve of step 2, converted from inches to cfs-hrs during the tabulation. The remaining columns contain self-explanatory computations. Columns 2 and 6 give the first working curve and columns 2 and 8 the second; they are plotted in Figure 17-9. Note that "lines of equal values" if drawn would be well above the working curves, therefore the routing intervals are adequately small. Also note that the second curve is shown only for the higher discharges in order to use a larger scale; ordinarily the entire curve is plotted.

Table 17-8 Working table for preparation of the working curve for Example 17-4.

O_2 (cfs)	$\frac{O_2}{2}$ (cfs)	S_2 (cfs-hrs)	$\frac{S_2}{\Delta t}$ (cfs)	$\frac{S_2}{\Delta t} + \frac{O_2}{2}$ (cfs)
(1)	(2)	(3)	(4)	(5)
0	0	0	0	0
50	25	70	140	165
150	75	164	328	403
300	150	248	496	646
800	400	651	1302	1702
1500	750	1302	2604	3354
3500	1750	3300	6600	8350
5000	2500	4540	9080	11580
7000	3500	5620	11240	14740
10000	5000	7130	14260	19260

Table 17-9 Operations table for the S-I method for Example 17-4.

Time (hrs)	$\bar{I}$ (cfs)	$\frac{S_2}{\Delta t} + \frac{O_2}{2}$ (cfs)	O (cfs)
(1)	(2)	(3)	(4)
0	0	0	0
.5	625*	625	285
1.0	1875	2215	1030
1.5	3125	4310	1880
2.0	4375	6805	2880
2.5	4615	8540	3610
3.0	3865	8795	3710
3.5	3125	8210	3450
4.0	2375	7135	3050
4.5	1635	5720	2440
5.0	900	4180	1810
5.5	265	2635	1210
6.0	0**	1425	630
6.5	0	795	375
7.0	0	420	160
7.5	0	260	82
8.0	0	178	53
etc.	etc.	etc.	etc.

* 625 cfs is the average discharge for the time from 0 to 0.5 hours, 1875 cfs the average discharge from 0.5 to 1.0 hours, and so on.

** Inflow ceases at 5.33 hours.

Table 17-10 Procedure for routing by the Storage-Indication method for Example 17-4.

Time (hrs)	$\bar{I}$ (cfs)	$\frac{S_2}{\Delta t} + \frac{O_2}{2}$ (cfs)	O (cfs)	Remarks
(1)	(2)	(3)	(4)	(5)
0	0	0	0	Given
.5	625			Given
		625		0 - 0 + 625 = 625
			285	From Figure 17-6
1.0	1875			Given
		2215		625 - 285 + 1875 = 2215
			1030	From Figure 17-6
1.5	3125			Given
		4310		2215 - 1030 + 3125 = 4310
			1880	From Figure 17-6
2.0	4375			Given
		6805		4310 - 1880 + 4375 = 6805
			2880	From Figure 17-6
etc.	etc.	etc.	etc.	etc.

6. Prepare the operations table.
Suitable headings and arrangement are shown in Table 17-12. Note that there is a column for instantaneous rates of inflow. These rates will be used for getting $\bar{I}$ values because it is difficult to select $\bar{I}$ values accurately enough from some portions of the plotted hydrograph.

7. Tabulate times and rates of inflow and compute $\bar{I}$ values.
Accumulated times are shown in column 1 of Table 17-12 at intervals of Δt = 0.5 days for the initial slow-rising portion of the PSH, at Δt = 0.1 days for the fast-rising and -falling portion, and again at Δt = 0.5 days for the slow recession. Instantaneous rates of inflow for those times are taken from the PSH of Figure 17-8 (or from column 4 of Table 21-7 if they are for the selected times) and shown in column 2. The $\bar{I}$ values of column 3 are arithmetic averages of entries in column 2.

8. Do the routing.
The procedure is the same as that given in Table 17-10 except when a change is made from one working curve to another. The changes are made as follows. At time 4.5 days the routing interval changes, therefore, the working curve must be changed. The outflow rate at that time is 116 cfs. Entering the second working curve with this rate gives 2,640 cfs as the value of $(S_2/\Delta t) + (O_2/2)$ in column 4 for the same time. Once this value is entered the routing continues with use of the second working curve. At time 6.0 days the routing interval changes back to the first one and therefore the first working curve must again be used. The outflow rate at that time is 357 cfs. Entering the first working curve with this rate gives 1,270 cfs as the value of $(S_2/\Delta t) + (O_2/2)$ in column 4 for that time. After entering this value the routing continues with use of the first working curve.

9. Determine the maximum storage attained in the routing.
The maximum storage attained in a reservoir during the routing of a single-peaked hydrograph occurs at the time when outflow equals inflow. The plotting in Figure 17-8 shows that this occurs at 5.33 days. For this time, Table 17-12 shows that O_2 = 364 cfs and $(S_2/\Delta t) + (O_2/2)$ = 6,480 cfs. Solving for S_2 gives $S_2 = \Delta t$ 6480 - $(O_2/2)$. With Δt = 0.1 days and O_2 = 364 cfs, S_2 = 0.1 6480 - (364/2) = 629.8 cfs-days, the maximum storage. To convert to AF use Equation 17-2, which gives 629.8/0.504 = 1,247 AF as the maximum storage in AF. To convert AF to inches use Equation 17-3 and the given drainage area of 8.0 square miles (see Example 17-1), which give 1247/53.3(8.0) = 2.93 inches as the maximum storage in inches. (Note: The storage can also be found by use of a storage-discharge curve or elevation-discharge and elevation-storage curves but with the Storage-Indication method it is generally best to use the above method.)

A comparison of peak rates of outflow shows that the mass-curve method of Example 17-1 gave a peak rate of 1.69 in./day, which converts to 363

Table 17-11 Working table for preparation of the working curves for Example 17-5.

Elevation (feet)	Discharge (O_2) (cfs)	Storage (S_2) (cfs-days)	$\frac{O_2}{2}$ (cfs)	For Δt = 0.5 days: $\frac{S_2}{\Delta t}$ (cfs)	For Δt = 0.5 days: $\frac{S_2}{\Delta t} + \frac{O_2}{2}$ (cfs)	For Δt = 0.1 days: $\frac{S_2}{\Delta t}$ (cfs)	For Δt = 0.1 days: $\frac{S_2}{\Delta t} + \frac{O_2}{2}$ (cfs)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
580.2	0	0	0	0	0	0	0
581.2	11.6	47.0	6	94	100	470	476
582.2	32.8	96.5	16	193	209	965	981
583.3	60.3	165	30	330	360	1650	1680
584.6	108	236	54	472	526	2360	2414
586.0	133	324	66	648	714	3240	3306
587.0	149	393	75	786	861	3930	4005
587.5	204	431	102	862	964	4310	4412
588.0	289	471	144	942	1086	4710	4854
588.5	353	512	176	1024	1200	5120	5296
590.0	365	643	182	1286	1468	6430	6612
592.0	382	832	191	1664	1855	8320	8511
595.0	401	1165	200	2330	2530	11650	11850

Table 17-12 Operations table for Example 17-5.

Time (days)	Inflow (cfs)	$\bar{I}$ (cfs)	$\frac{S_2}{\Delta t} + \frac{O_2}{2}$ (cfs)	Out-flow (cfs)
(1)	(2)	(3)	(4)	(5)
0	0	0	0	0
.5	70	35	35	3
1.0	79	74	106	12
1.5	84	82	176	28
2.0	88	86	234	38
2.5	99	94	290	48
3.0	110	104	346	57
3.5	128	119	408	72
4.0	156	142	478	94
4.5	245	200	584*	116
			2640**	116
4.6	269	257	2781	119
4.7	308	288	2950	123
4.8	380	344	3171	129
4.9	522	451	3493	137
5.0	2002	1262	4618	240
5.1	1049	1526	5904	359
5.2	577	813	6358	363
5.3	393	485	6480	364
5.4	312	352	6468	364
5.5	267	290	6394	363
5.6	217	242	6273	362
5.7	200	208	6119	361
5.8	184	192	5950	360
5.9	174	179	5769	358
6.0	164	169	5580**	357
			1270*	357
6.5	138	146	1059	266
7.0	118	128	921	175
7.5	106	112	858	148
8.0	94	100	810	142
etc.	etc.	etc.	etc.	etc.

* From first working curve.
**From second working curve.

cfs, and the Storage-Indication method gave 364 cfs, which is excellent agreement. But a comparison of maximum storage in inches shows that the mass-curve method of Example 17-1 gave 2.82 inches, the graphical mass-curve method of Example 17-3 gave 2.80 inches, and the Storage-Indication method gave 2.93 inches. The discrepancy is for the most part due to use of small-scale graphs for the working curves. Larger graphs would reduce the discrepancy.

Storage-Indication Method as Used in the SCS Electronic Computer Program.- SCS electronic computer program for watershed evaluations uses the Storage-Indication method only for reservoir routings. The chief difference between the manual procedure of Example 17-5 and the electronic-computer procedure is that in the latter no working curves are used. Instead, the working equation is solved during a process in which interpolations are made in the elevation-discharge and elevation-storage data stored in the computer. The process is repeated during the routing just as the working curve is repeatedly used in manual routing. The machine routing has a numerical accuracy greater than that of the manual routing, but the machine cannot improve the accuracy of the input data. Details of the machine routing process are given in pages A-61 through A-66 of the report titled "Computer Program for Project Formulation - Hydrology," by C-E-I-R, Inc. Arlington, Va., January 1964, which was prepared for SCS. Copies of this report are available from the Washington, D. C. office of SCS.

Culp's Method.- Some routing methods are developed for solving special problems, for which they have a high efficiency. One such method is described next.

In the design of an emergency spillway of a dam it is SCS practice to base the design on the results from a routing of an Emergency Spillway Hydrograph. Because all of the spillway dimensions cannot be known in advance, it is necessary to route the hydrograph through three or four different spillways with assumed dimensions before the spillway with the proper dimensions can be found. M. M. Culp's routing method eliminates much of that work by giving the routed peak discharge without the use of spillway dimensions. The following example shows an application of the method to the structure used in previous examples. The example is lengthy because many details are given; after the method is understood it will be seen to be fast and easy to apply.

Example 17-6.--Find the routed peak discharge to be used in design of an emergency spillway for the structure of Example 17-1. The required difference in elevation between the crest of the spillway and the reservoir water surface, H_p, is 4.0 feet during the peak discharge. Watershed and structure data are given in examples 17-1 and 17-2.

1. Prepare the elevation-discharge curve for the principal spillway.

   This curve was prepared for Example 17-1 with the discharges in inches per hour. It will be used here as shown in Figure 17-10(a) with discharges in cfs.

2. Prepare the elevation-storage curve for the structure.
This curve was prepared for Example 17-1. Only the portion above the sediment storage will be used here; it is shown in figure 17-10(a).

3. Determine the elevation of the emergency spillway crest.
According to SCS criteria, the elevation of the emergency spillway crest can be at or above the maximum water-surface elevation attained in the reservoir during the routing of the Principal Spillway Hydrograph (PSH) or its mass curve (PSMC). The water-surface elevation found in Example 17-1 will be used here as the crest elevation. This elevation is 589.5 feet with floodwater storage of 2.82 inches.

4. Determine the water-surface elevation of the floodwater remaining in the reservoir after 10 days of drawdown from storage at the water-surface elevation attained in routing the PSH or PSMC.
This step is required by SCS criteria. The determination is made in Example 17-2 and those results will be used here. The water-surface elevation after 10 days of drawdown is 581.1 feet with floodwater storage at 0.20 inches.

5. Prepare the Emergency Spillway Hydrograph (ESH) and its mass curve (ESMC).
The ESH for this example was prepared using the method of Example 21-5 and the following data: drainage area = 8.0 square miles, time of concentration = 2.0 hours, runoff curve number = 75, design storm rainfall = 9.1 inches, storm duration = 6.0 hours, runoff = 6.04 inches, hydrograph family = 2, T_o = 5.05 hours, initial T_p = 1.4 hours, T_o/T_p = 3.61, selected T_o/T_p = 4, revised T_p = 1.26 hours, q_p = 3,073 cfs, and $Q(q_p)$ = 18,560 cfs. The ESMC was prepared using Table 21-17 and the following data: hydrograph family = 2, T_o/T_p = 4, T_p = 1.26 hours, and Q = 6.04 inches. The hydrograph is shown in Figure 17-10(b) and the mass curve in Figure 17-10(c).

(Note: The above steps are taken, in much the same way, regardless of which manual method of routing is used for this kind of problem. The following steps apply to the Culp method.)

6. Determine the time at which the emergency spillway begins to flow during passage of the ESH or ESMC.
For this example the time was found by routing the ESMC of step 5 by the method of Example 17-1, using the curves of Figure 17-10(a) as working curves. The routing was started with 0.20 inches of floodwater in the reservoir (SCS criteria require the ESH or ESMC routing to start at the elevation for the floodwater remaining after the 10-day drawdown period; see step 4). The emergency spillway began to flow at 2.9 hours, at which time the mass outflow was 0.06 inches. The time and outflow are indicated by point c1 on Figure 17-10(c)

7. Determine the average discharge of the principal spillway during passage of the ESH or ESMC through the emergency spillway. The principal spillway average discharge is for the period during which the reservoir storage rises from the elevation of the emergency spillway crest to the crest elevation plus H_p. Use the elevation-discharge curve of Figure 17-10(a) to find the discharges at the two elevations. These discharges are 361 and 392 cfs respectively; their average is 376 cfs.

8. Locate a reference point in the ESH for use in later steps. The reference point, shown as point b1 in Figure 17-10(b), is located at the time determined in step 6 and at the average discharge determined in step 7. A second point, not actually necessary in the work, is shown as b2 on the recession side. A straight line connecting points b1 and b2 represents the principal spillway outflow rate during the period used in step 7.

9. Compute the slope of the principal spillway mass outflow line for use on the mass inflow graph. The mass outflow to be used is for the period considered in step 7. Full pipe flow occurs and the mass outflow is adequately represented by a straight line. The slope of the line for this example must be in inches per hour because the mass inflow scales are for inches and hours. To get the slope, convert the average discharge of step 7 by use of Equation 17-5, which gives 376/645(8.0) = 0.073 inches per hour.

10. Plot a reference line and a working line of principal spillway mass outflow on the graph for mass inflow. The lines are for the period considered in step 7 but for working convenience they are extended beyond the limits of the period. To plot the reference line, first locate point c2 on the mass inflow curve of Figure 17-10(c) at the time determined in step 6, then through c2 draw a straight line having the slope determined in step 9; this gives line A as shown. To plot the working line, first determine the storage associated with H_p, which is 1.84 inches as shown in Figure 17-10(a), then draw line B parallel to line A and 1.84 inches of runoff above it as shown in Figure 17-10(c).

11. Find the period within which the emergency spillway peak discharge will occur. Point c3 is at the intersection of the mass inflow curve and line B in Figure 17-10(c). Locate point b3 on the ESH of Figure 17-10(b) at the time found for c3. Points b3 and b2 are the end points for the period within which the emergency spillway peak discharge will occur.

12. Select several working discharges between points b3 and b2. Four selected working discharges are indicated by points b4, b5, b6, and b7 in Figure 17-10(b); the discharges are 4,750, 3,500, 2,200, and 920 cfs respectively. These discharges represent the peak discharges of outflow hydrographs.

(Note: After some experience with this method, it may be found easier to select only two working discharges in this step, to work through steps 13 to 15, and if the results are unsatisfactory to return to step 12 again by selecting a third working discharge, working through steps 13 through 15 for that discharge, and so on.)

13. Compute a volume-to-peak for each working discharge of step 12.
In the Culp method the rising side of the outflow hydrograph for a trapezoidal spillway is taken as being nearly parabolic so that the volume from the beginning of rise to the peak rate, or the volume-to-peak, is:

$$Qe = 0.62\ (qe - qps)\ Te \qquad \text{(Eq. 17-18)}$$

where Qe is the volume in cfs-hrs, qe is the working discharge of step 12 in cfs, qps is the principal spillway rate of step 7 in cfs, and Te is the time in hours from point b1 to the peak time. The volume Qe must be converted to a unit usable with the mass inflow curve, in this case, inches. The summary of work for this step is given in Table 17-13. In the columns for points b4 through b7, the items in line 1 are from step 12; items in line 2 are from step 7; items in line 3 are obtained by subtracting qps from qe; items in line 4 are obtained by inspection of Figure 17-10(b); items in line 5 are products of (qe - qps) x Te; items in line 6 are products of (Qe/0.62) x 0.62; items in line 7 are Qe's of line 6 divided by the drainage area of 8.0 square miles; items of line 8 are Qe's of line 7 divided by 645. Each Qe of line 8 applies only at the time indicated by its point on the ESH.

14. Plot a curve of mass inflow minus mass outflow.
This is a working curve, not the complete curve of inflow minus outflow. Subtract each Qe of line 8, Table 17-13, from the inflow amount at the identical time on the mass inflow curve of Figure 17-10(c) and plot the result as shown for points c4, c5, c6, and c7. Connect the points with a curve, line C.

15. Determine the time and rate for the emergency spillway peak discharge.
The intersection of lines B and C, at point c8 in Figure 17-10(c), gives the time at which the emergency spillway peak discharge occurs The total discharge rate at that time is 3,050 cfs as shown by the corresponding point b8 on the ESH of Figure 17-10(b). The emergency spillway discharge rate is 3050 - 376 = 2,674 cfs, which occurs when the reservoir water surface is at the given elevation of 593.5 feet (crest elevation plus H_p). This step completes the routing. Design of the emergency spillway now follows with use of ES-98, ES-124, and spillway criteria.

If H_p is not known in advance, the Culp method can be used with assumed values of H_p to get associated discharges from which the suitable combination of H_p and discharge can be selected. For earth spillways H_p can be closely approximated from permissible velocities and the appropriate

Table 17-13 Working table for Culp method step 13 of Example 17-6.

Line	Item	Unit	Point: b4	b5	b6	b7
1	qe	cfs	4750	3500	2200	920
2	qps	cfs	376	376	376	376
3	qe - qps	cfs	4374	3124	1824	544
4	Te	hrs	2.1	2.8	3.4	4.1
5	Qe/0.62	cfs-hrs	9180	8750	6200	2230
6	Qe	cfs-hrs	5680	5420	3840	1380
7	Qe	csm-hrs	710	677	480	173
8	Qe	in.	1.102	1.050	0.745	0.268

length and chosen profile of the inlet channel. A close approximation of the emergency spillway discharge rate can be obtained in this way for an H_p value near the middle of the desired range to get a "C curve" (line C on Figure 17-10(c)). The average discharge in the conventional drop inlet under full pipe flow conditions varies only slightly as H_p varies relatively greatly, thus the discharge through the emergency spillway can be closely approximated from such an average C curve. If refinement is justified, then trial adjustments on the slope of line B will give the required accuracy. The correction process converges rapidly. For preliminary layouts or comparative cost studies such refinement is seldom justified.

Short-Cuts for Reservoir Routings.- Various equations and charts have been developed for quickly estimating the required storage in a reservoir or the required capacity of a spillway, such estimates being used in preliminary studies of structures or projects. The equations and charts are usually based on the results of routings so that using the equation or chart is in effect a form of routing.

A typical short-cut is the graph, Figure 17-11. The curve through the circled points is based on information in table 2 on page 39 of "Low Dams," a design manual prepared by the Subcommittee on Small Water Storage Projects, National Resources Committee, Washington, D. C., 1938 (the manual is out of print and no longer available for purchase). Relationships of this kind are developed from routings made through a particular type of spillway and they apply only to that type. The form of standard inflow hydrograph used for routing also affects the relationship and the same form must be applicable when the short-cut is used. With such a relationship if any three of the four variables are known the fourth can be estimated. Usually either the reservoir storage or the reservoir discharge rate is the unknown.

The triangular point on Figure 17-11 is for the routing made in Example 17-6. For that example the outflow/inflow ratio is 3050/10200 = 0.30 and the storage/inflow-volume ratio is 2.82/(2.62 + 1.84) = 0.63. Note that the emergency spillway "surcharge" storage is included when computing the volume ratio. The cross points, for "miscellaneous routings", are for routings of several kinds of hydrographs through emergency spillways of the SCS type. The "Low Dams" curve appears to be an enveloping curve for the points. As such it can be used for making conservative estimates. Thus, if the inflow volume is 8.15 inches of runoff and the total available storage is 5.7 inches then the storage ratio is 0.7; at that ratio the discharge ratio is 0.4, which means that the peak outflow rate will be not more than 0.4 of the peak inflow. Such estimates are often useful in preliminary work.

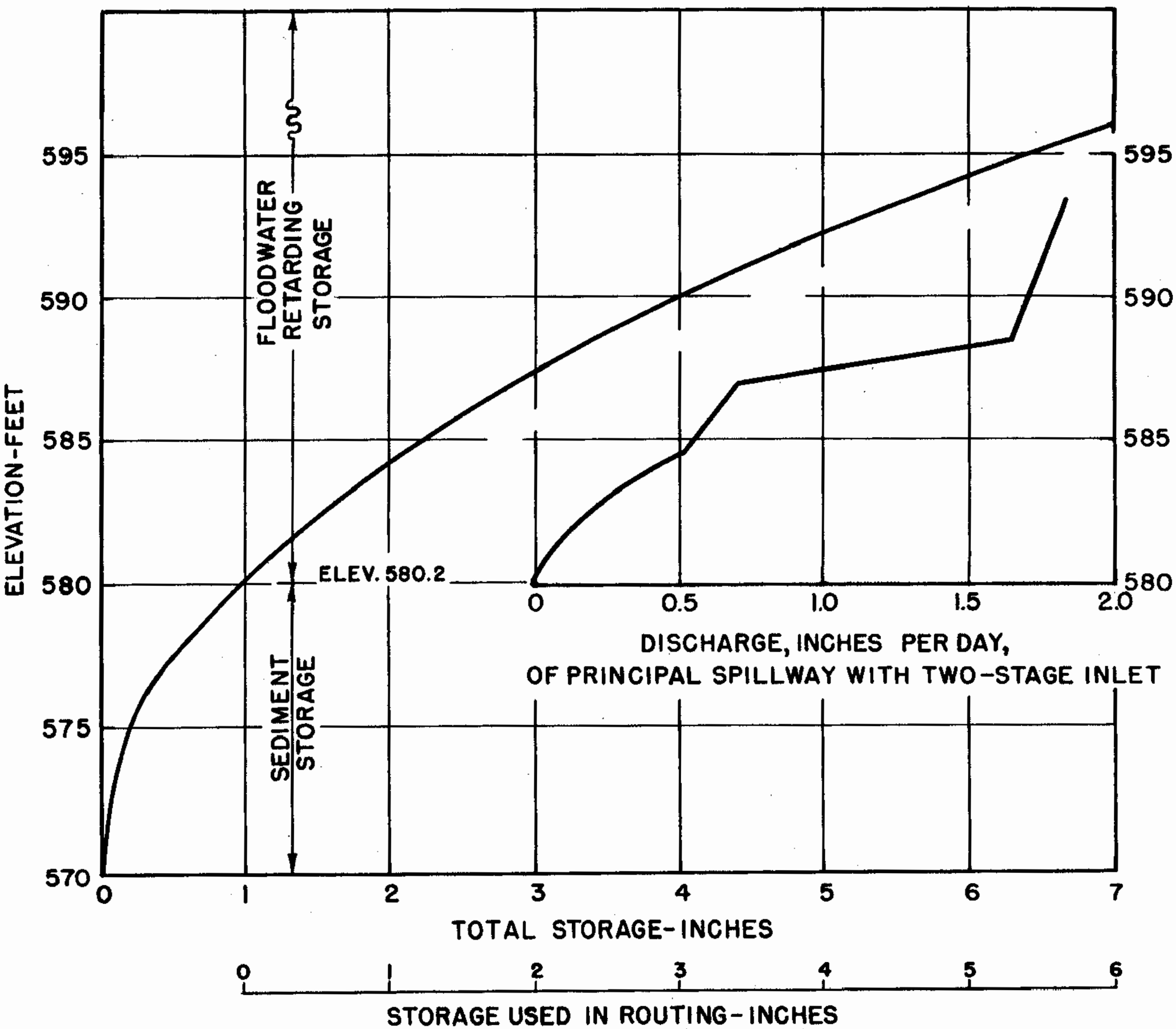


Figure 17-1. Elevation, storage, discharge relationship for a reservoir.

NEH Notice 4-102, August 1972

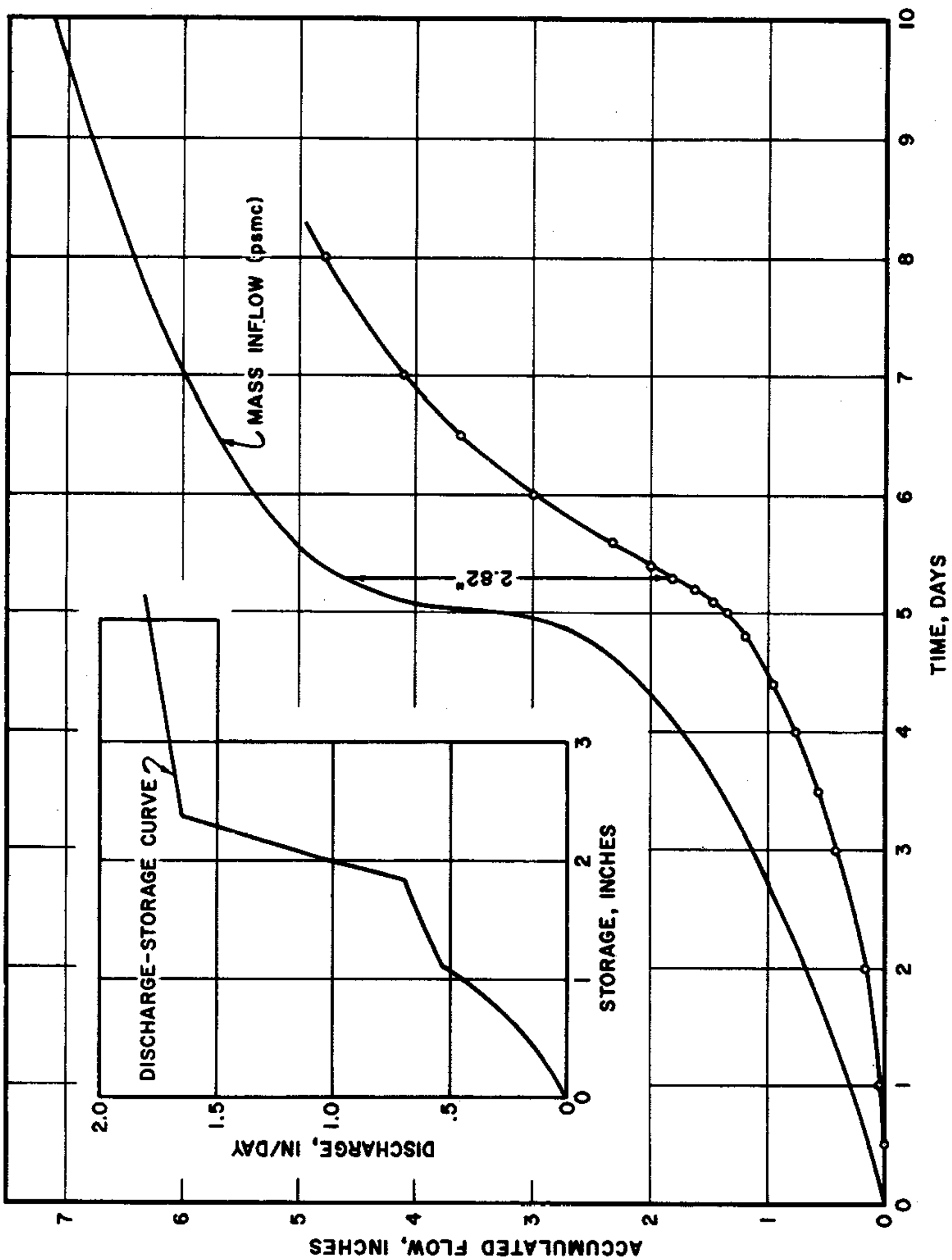


Figure 17-2. Storage, discharge relationship and plotted mass inflow curve for a reservoir.

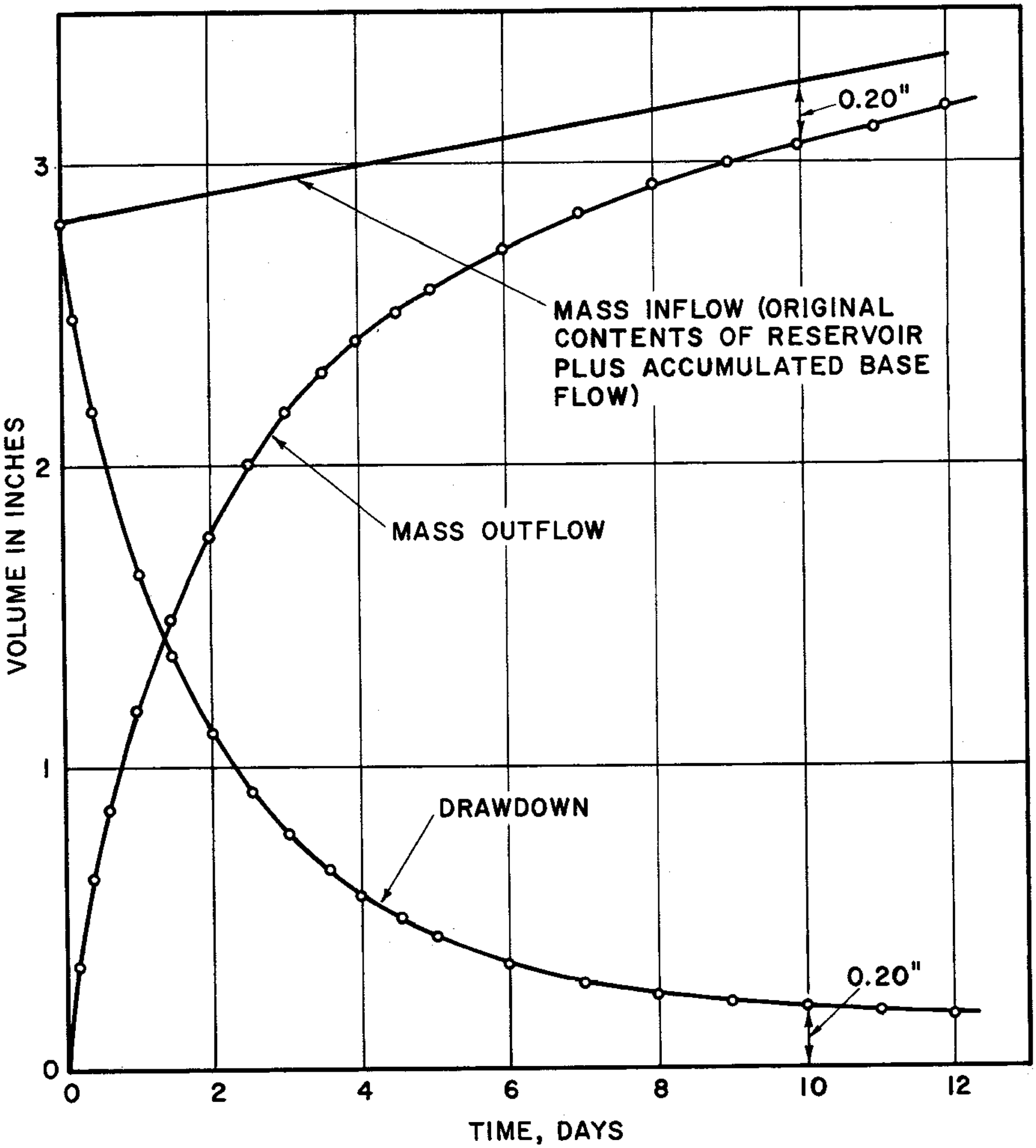


Figure 17-3. Mass inflow, storage, and mass outflow curves for Example 17-2.

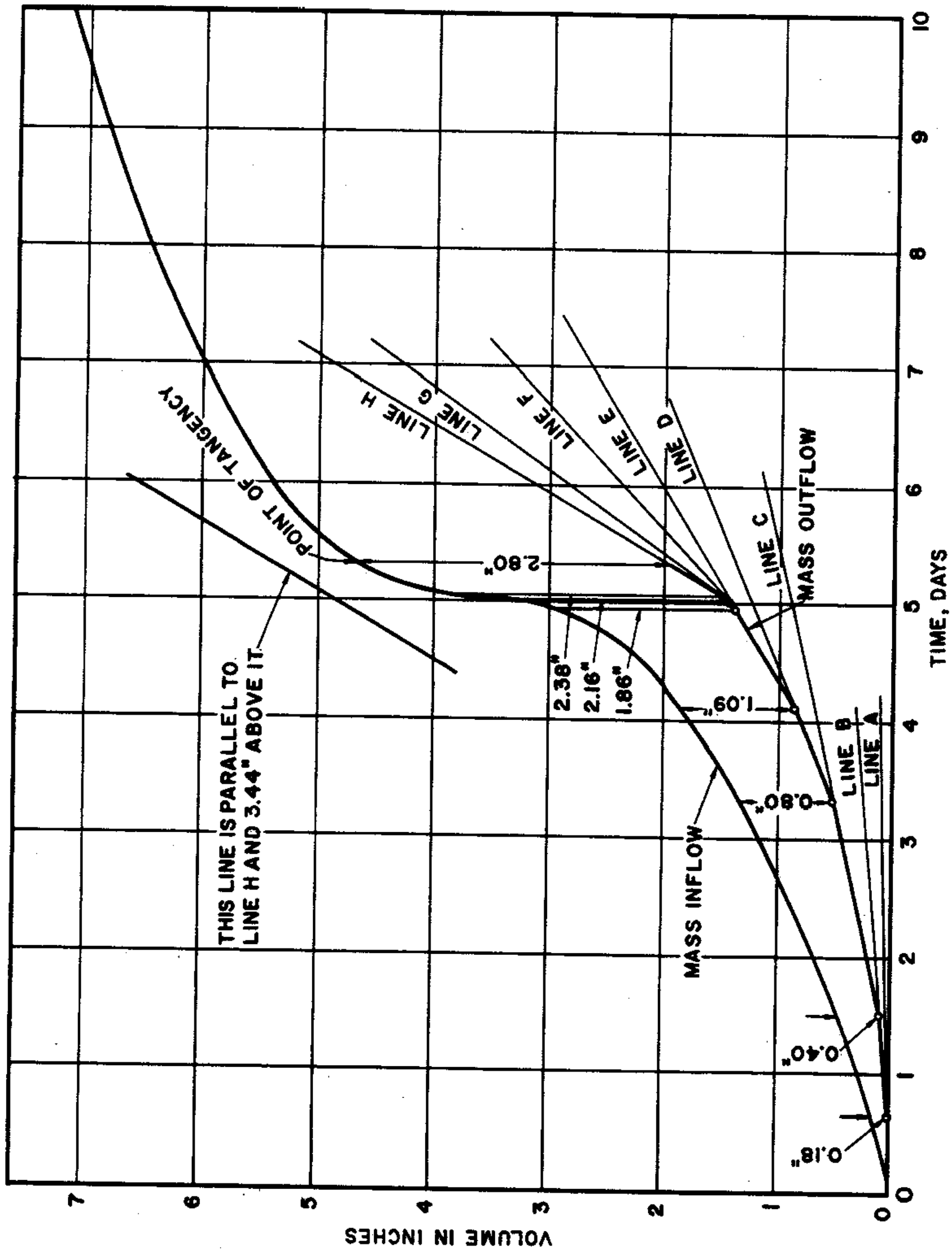


Figure 17-4. Graphical version of Mass Curve method of reservoir routing for Example 17-3.

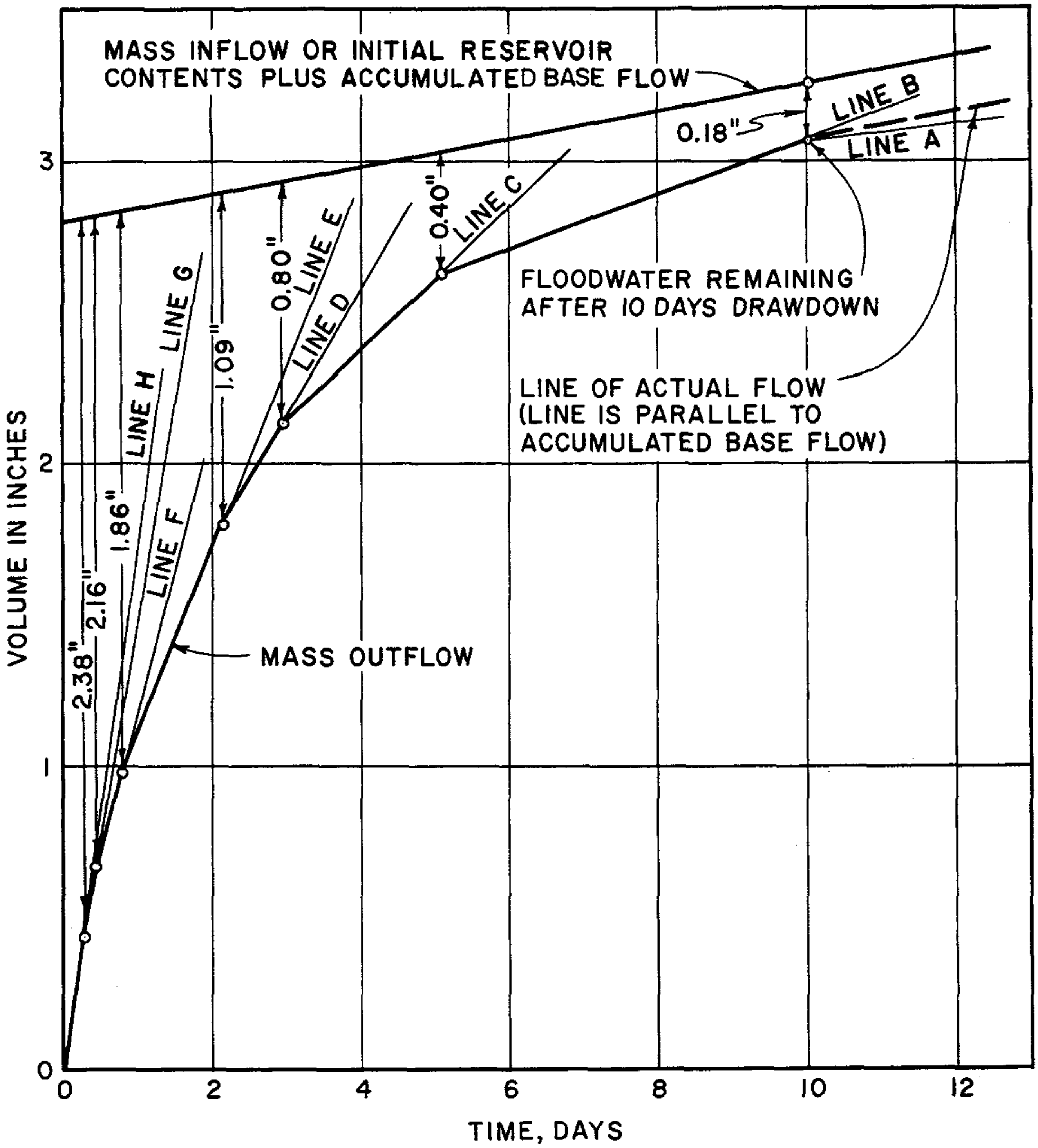


Figure 17-5. Graphical version for Example 17-2, Step 4.

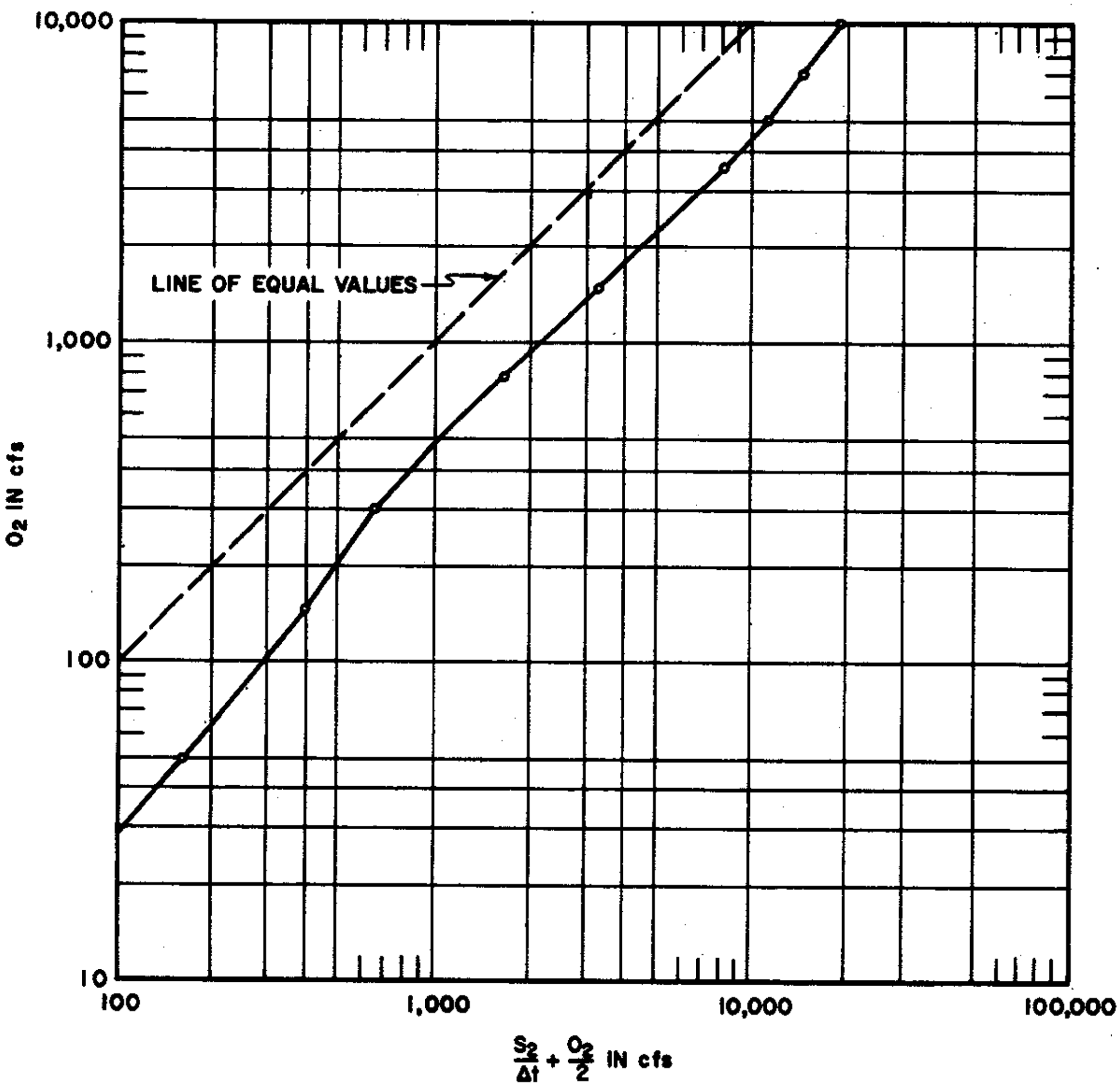


Figure 17-6. Working curve for Storage-Indication method of reservoir routing for Example 17-4.

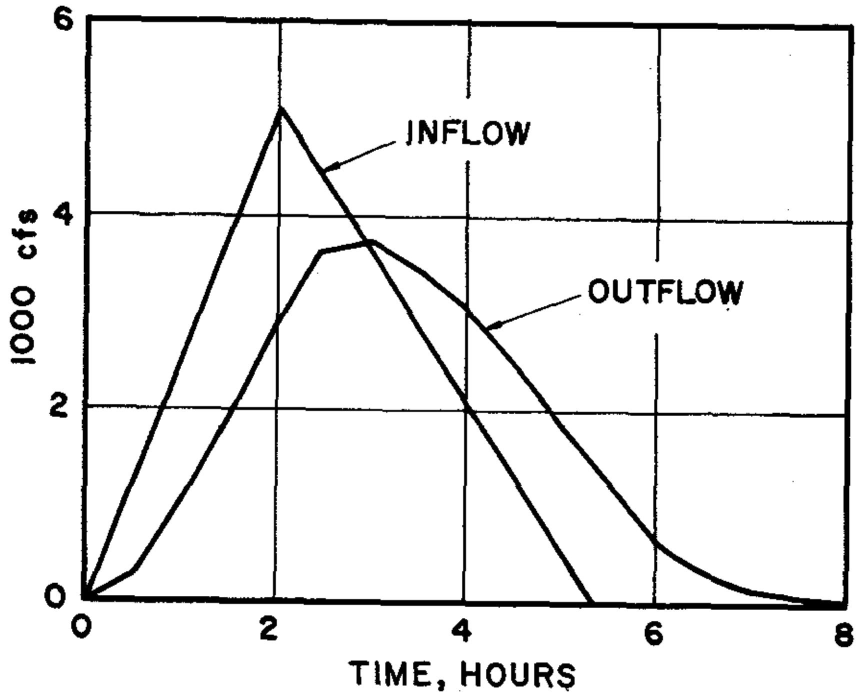


Figure 17-7. Inflow and outflow hydrograph for Example 17-4.

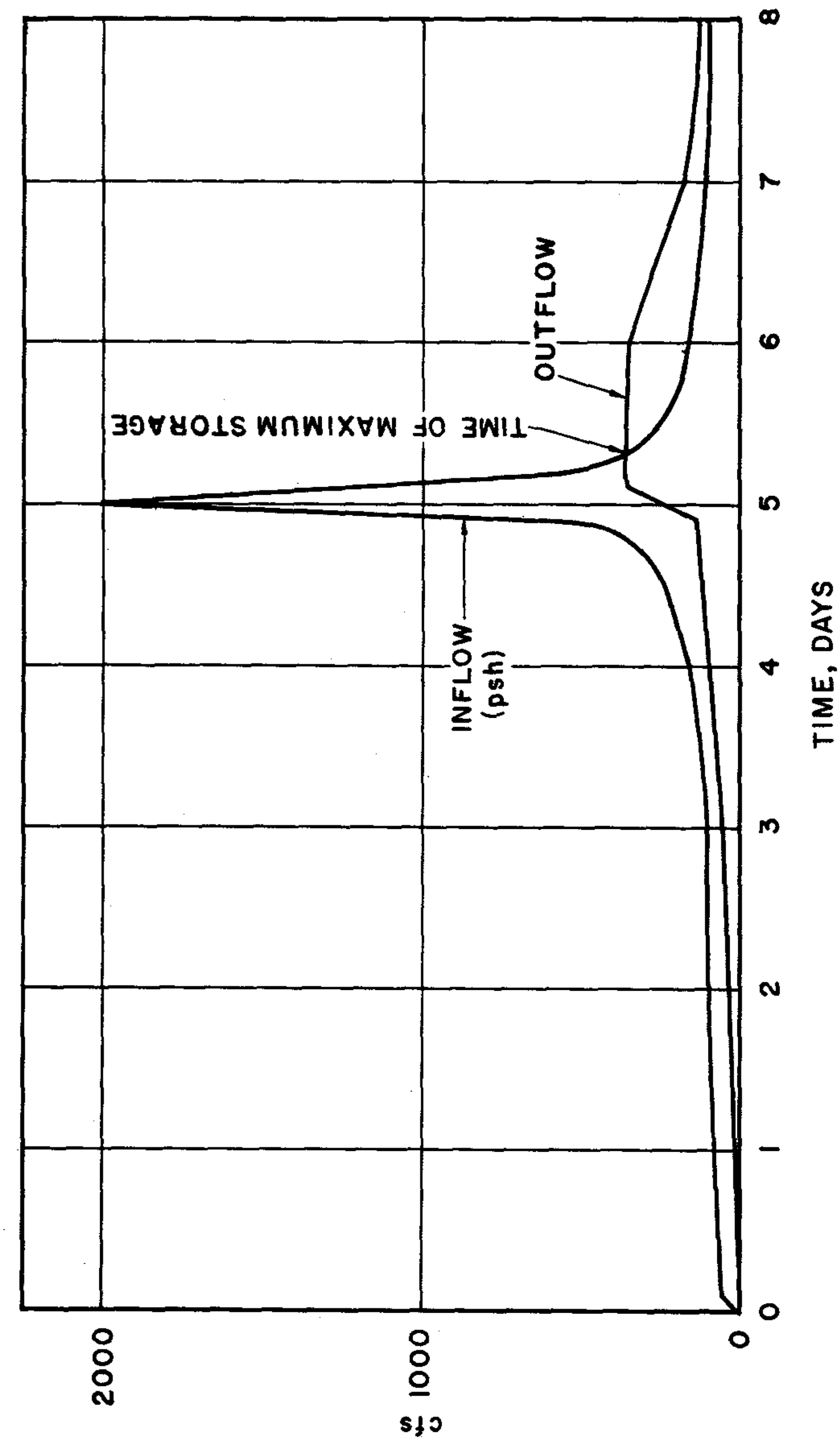


Figure 17-8. Principal spillway hydrograph and outflow hydrograph for Example 17-5.

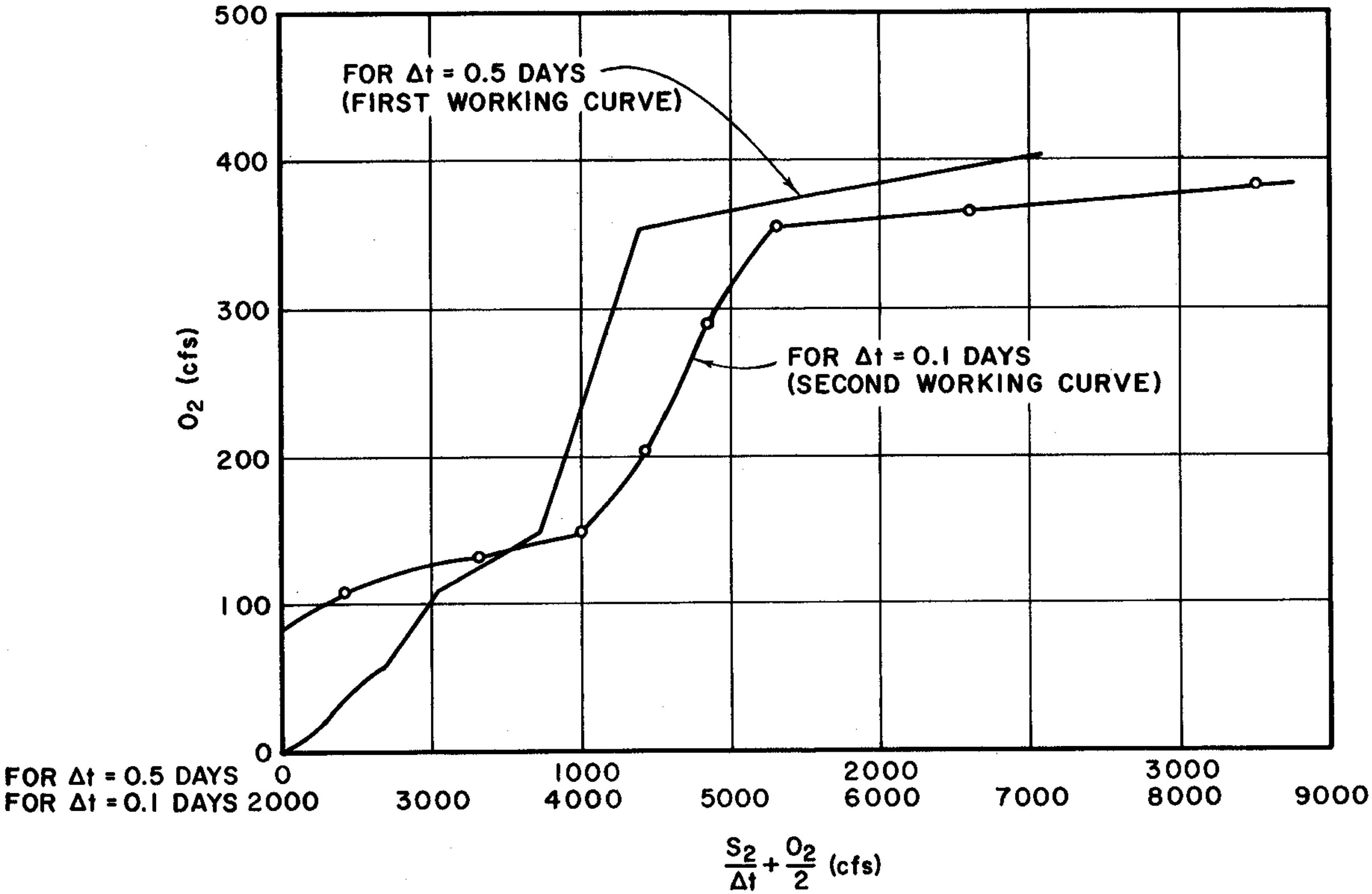


Figure 17-9. Working curves for Storage-Indication method of reservoir routing for Example 17-5.

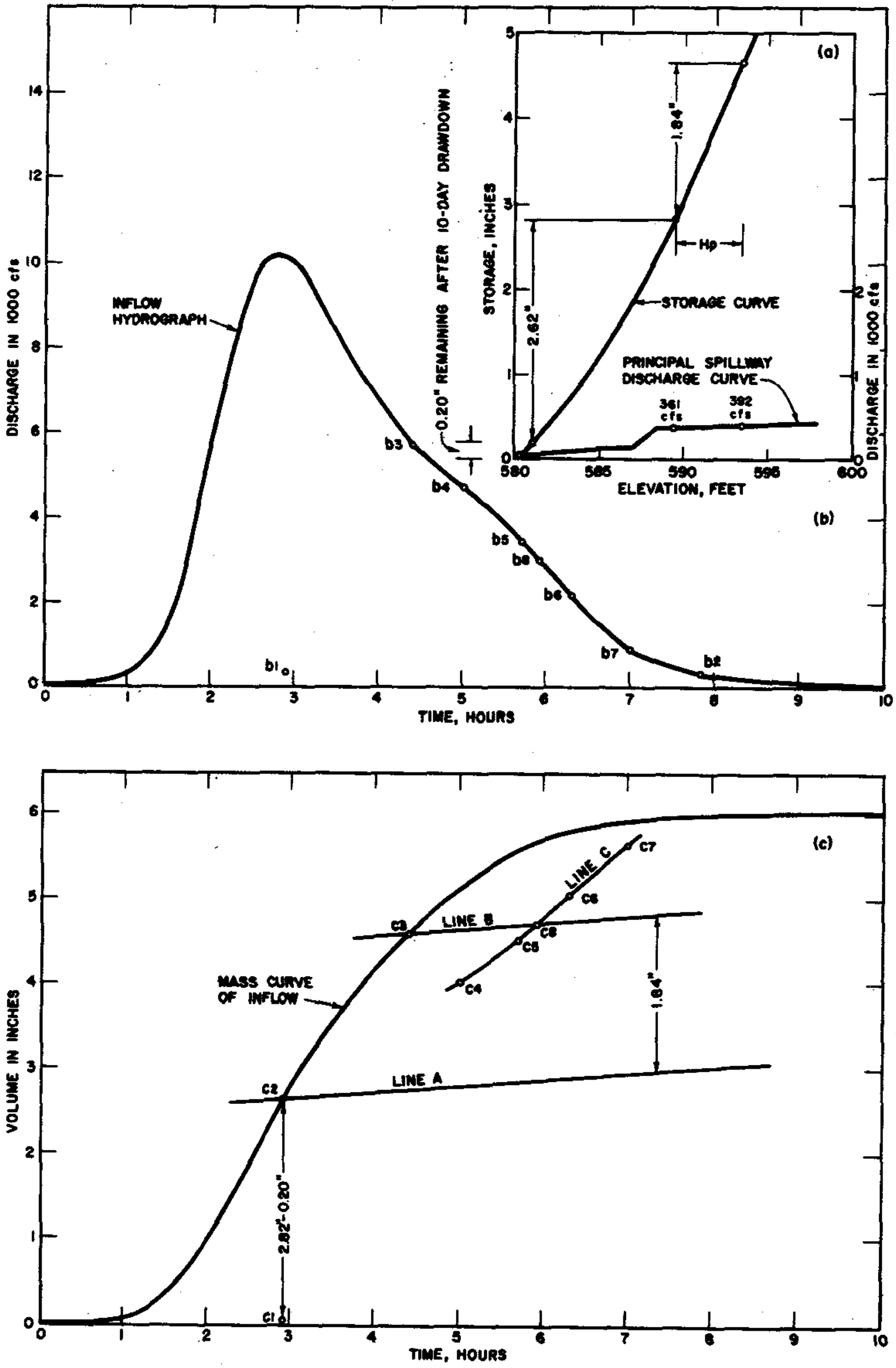


Figure 17-10. Culp's method of reservoir routing for Example 17-6.

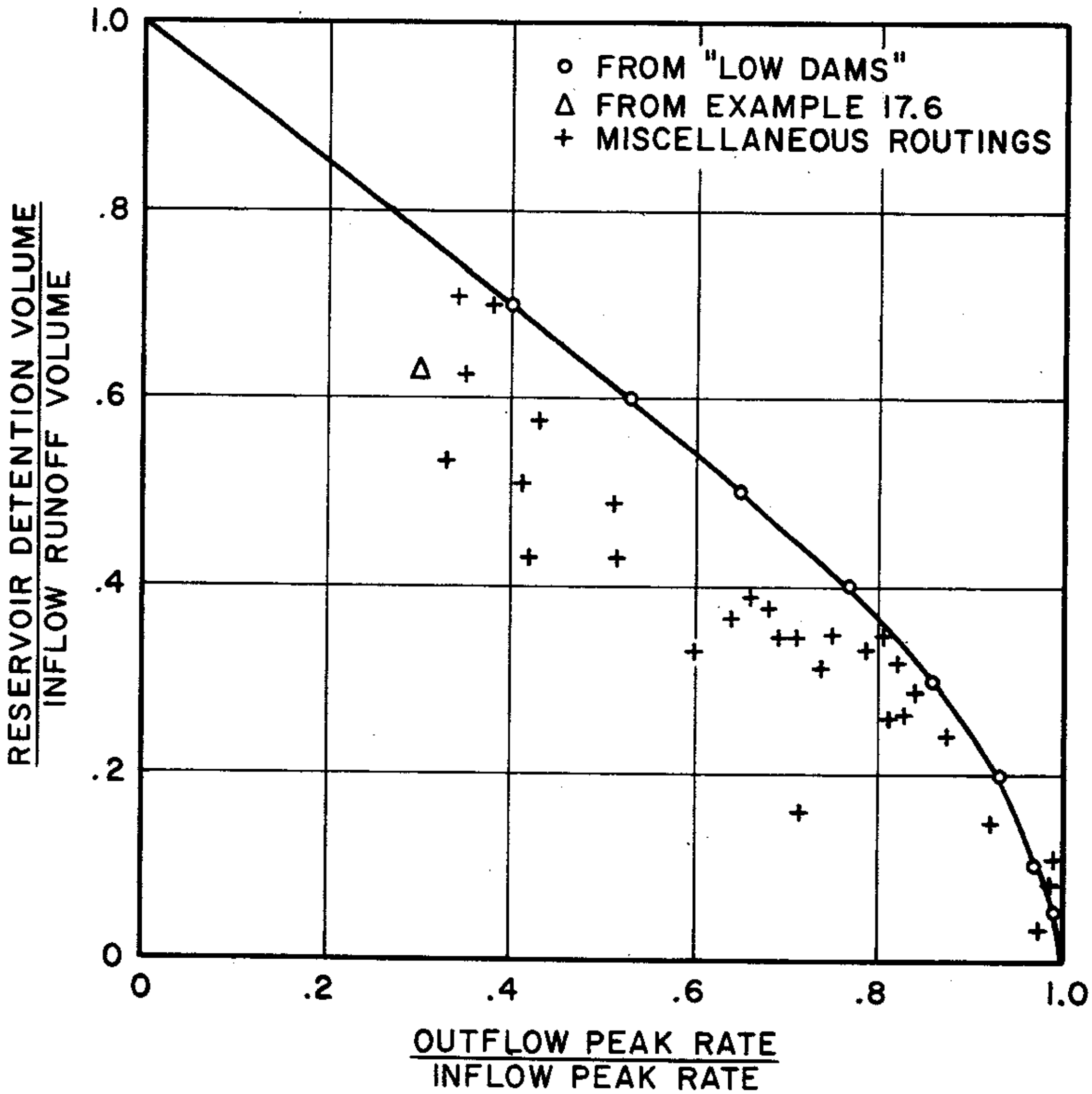


Figure 17-11. Typical shortcut method of reservoir flood routing.

Channel Routing Methods

The Convex method of routing through stream channels is presented in this part. The method is derived from inflow-outflow hydrograph relationships and, because of this, the method has some features not possessed by channel routing methods derived from consideration of the continuity equation. The Storage-Indication method of channel routing, presented in Example 17-4, will not be discussed here, but discussions of procedures for adding local inflows, deducting transmission losses, and routing through stream systems also apply to that method.

Theory of the Convex Method

The Convex method is based on the following principle: When a natural flood flow passes through a natural stream channel having negligible local inflows or transmission losses, there is a reach length L and a time interval Δt such that O_2 is not more than the larger nor less than the smaller of the two flows I_1 and O_1. Δt is considered as both the travel time of the flood wave through the reach measured at the beginning of the rising portion of the hydrograph at both ends of the reach; and the required routing time interval.

The principle requires that:

$$\text{If } I_1 \geq O_1, \text{ then } I_1 \geq O_2 \geq O_1 \qquad \text{(Eq. 17-19)}$$

$$\text{If } I_1 \leq O_1, \text{ then } I_1 \leq O_2 \leq O_1 \qquad \text{(Eq. 17-20)}$$

In general, inequality Equation 17-19 applies to rising portions of hydrographs and Equation 17-20 to falling portions. Note that I_2 does not enter into the principle; this makes the Convex method a forecasting method (see under "Discussion").

The routing principle can be extended to include local inflows and transmission losses but this unnecessarily complicates the working equation. It is common practice to add local inflows to the routed outflow hydrographs to get the total outflows, and this practice will be followed here. There may be situations, however, in which the local inflow is added to the inflow hydrograph and then routed. Small transmission losses are generally deducted after the routing, large ones during the routing; for a discussion of transmission losses see the heading "Effects of transmission losses on routed flows."

The routing or working equation is formed after examination of typical inflow and outflow hydrographs such as those in Figure 17-12. Typical flood wave combinations of I_1, O_1 and O_2 are shown on the rising and falling sides of the hydrographs. The routing principle states that for a properly selected reach length L, hence Δt, O_2 will fall somewhere on or between I_1 and O_1 in magnitude but not above or below them. This is evident on Figure 17-12 despite the displacement of O_2 in time; it is the magnitudes that are of concern here.

The next step is to recognize that I_1, O_1 and O_2 are members of a Convex set[1/]. For such a set, if points A and B are in the set then all points on a straight line connecting A and B are also in the set. Because the concern is with magnitudes and not with time it is not necessary for O_2 to be physically on the line between I_1 and O_1. The routing equation can now be written based on the theory of convex sets. For the situation just described, and using proportions as shown on the inset of Figure 17-12, the routing or working equation is:

$$O_2 = (1-C)O_1 + C\,I_1 \qquad \text{(Eq. 17-21)}$$

where C is a parameter with the range:

$$\text{zero} \leq C \leq \text{one} \qquad \text{(Eq. 17-22)}$$

Given Equation 17-22, Equation 17-21 meets the requirements of Equations 17-19 and 17-20 and therefore of the routing principle.

The routing method based on Equation 17-21 is called the Convex method to call attention to the equation's background.

It follows from Equation 17-21 that:

$$C = \frac{O_2 - O_1}{I_1 - O_1} \qquad \text{(Eq. 17-23)}$$

In the inset of Figure 17-12 the relationships between I_1, O_1, and O_2 make similar triangles, so that:

$$\frac{O_2 - O_1}{\Delta t} = \frac{I_1 - O_1}{K} \qquad \text{(Eq. 17-24)}$$

where K is considered the reach travel time for a selected steady flow discharge of a water particle through the given reach. From Equation 17-24 it follows that:

$$\frac{\Delta t}{K} = \frac{O_2 - O_1}{I_1 - O_1} \qquad \text{(Eq. 17-25)}$$

Combining Equations 17-23 and 17-25 gives:

$$C = \frac{\Delta t}{K} \qquad \text{(Eq. 17-26)}$$

1/ Enough of the theory of convex sets for the purposes of this chapter is given in pages 41-42 of "An Introduction to Linear Programming," by A. Charnes, W. W. Cooper, and A. Henderson; John Wiley and Sons, Inc., New York, 1953.

from which comes the equation that defines Δt, the wave travel time and also the required routing interval:

$$\Delta t = C\,K \qquad \text{(Eq. 17-27)}$$

Discussion

This much of the theory is enough for making a workable routing method. The emphasis in this chapter is on working examples, not on theory, therefore the additional results from the theory are summarized in the next section without giving derivations or proofs. Further work can be done on some aspects of the Convex routing method but even in its present state the method is highly useful for most types of problems of routing flood flows through stream channels.

The theory as given so far can be used for exploratory routings by assuming magnitudes for any two of the variables in Equation 17-27 computing the third, and using Equation 17-21 with various inflow hydrographs. Such routings show the features of the Convex method. In Figure 17-12, for example, note that outflow begins at one routing interval, Δt, after inflow begins, which is to be expected for a stream reach because it takes water waves time to travel through the reach. It is chiefly this characteristic that distinguishes the Convex method from channel methods based on the continuity equation. In Convex routing the peak rate of the outflow hydrograph does not fall on the recession limb of the inflow hydrograph, as in reservoir methods. But, as in all routing methods, the maximum storage in the reach is attained when outflow equals inflow (at point A in Figure 17-12). The maximum storage is represented by the area under the inflow hydrograph to the left of point A minus the area of the outflow hydrograph to the left of point A. Also note that inflow I_2 does not appear in the working equation though it does appear in equations for other channel methods. This feature makes the Convex method a forecasting method. For example, if the routing interval is one day, today's inflow and outflow are known and local inflow is known or negligible, then tomorrow's outflow can be predicted accurately without knowing tomorrow's inflow. The predictive feature is more important for large rivers than for small streams because the routing interval for reaches of such streams is usually short.

Some Useful Relationships and Procedures

Of the equations so far given, only Equations 17-21 and 17-27 are needed in practical applications of the Convex method. The first is the working equation and the second an auxiliary equation used once before a routing begins. Several other relationships and procedures also useful in applications follow.

Determination of K. - K is the reach travel time for a selected steady-flow discharge and can be computed using Equation 17-7 substituting K for T_t. Example 17-8 shows a preferred method for selecting the discharge. The K used in the Muskingum routing method (refs. 2 and 3) may also be used as the K for the Convex method.

<u>Determination of C.</u> - From Equations 17-17 and 17-26 the parameter or routing coefficient C can be derived as the ratio of two velocities: that is, $C = V/U$, where V is the steady-flow water velocity related to the reach travel time for steady flow discharge, K, and U is considered the wave velocity related to the travel time of the wave through the reach, Δt. For practical purposes C may be estimated from an empirical relationship between C and V shown in Figure 17-13. The dashed line in the Figure is represented by the equation:

$$C = \frac{V}{V + 1.7} \qquad \text{(Eq. 17-28)}$$

In some applications it is more convenient to use Equation 17-28 than Figure 17-13. The "x" used in the Muskingum routing method (refs. 2 and 3) may also be used to approximate C. The approximation is:

$$C \approx 2x \qquad \text{(Eq. 17-29)}$$

In the Muskingum procedure the x is sometimes determined only to the nearest tenth; if this is done then C is approximated to the nearest two tenths and accurate routing results should not be expected.

<u>Determination of Δt.</u> - If C and K are known, from Equation 17-27, there is only one permissible routing interval. This permissible interval may be an inconvenient magnitude because it is either an unwieldy fraction of an hour or does not fit the given hydrograph. In selecting a suitable routing interval keep in mind that too large an interval will not accurately define the inflow hydrograph and that too small an interval will needlessly increase the effort required for the routing. A generally suitable rule of thumb to follow is that the selected routing time interval, Δt^*, should be no greater than 1/5 of the time from the beginning of rise to the time of the peak discharge of the inflow hydrograph, or:

$$\Delta t^* \leq \frac{T_p}{5} \qquad \text{(Eq. 17-30)}$$

where T_p is the time to peak (Chapter 16). If the hydrograph has more than one peak the interval should be selected using the T_p for the shortest of the rise periods of the important peaks. It is important that an end-point of a time interval fall at or near the inflow peak time and any other large change in rate.

* *

<u>Procedure for routing through any reach length.</u>- The relationship of K, C, and Δt is valid for one and only one routing reach length for a given time interval and inflow hydrograph. If Δt is to be changed to Δt^* (desired routing time interval) it follows from Equation 17-27 that either (1) C or K must be changed (Method 1) or, (2) routing through a series of subreaches, L^*, (Equation 17-32) must be made until the sum of the travel time of the Δt's for each subreach, L^*, equal the desired travel time, Δt^*, for the total reach, L (Method 2). Selection of either method

depends on the manner of computation and the consistency of the answers desired. Method 1 may be used when rough approximations of the routing effect are desired and manual computation is used. Method 2 is used when consistency of the routing is important or a computer is used. Consistency, as used here, refers to the changes in the outflow hydrograph (T_p and q_p) caused by varying Δt^*. If there is little change in the hydrograph when Δt^* is changed the routing is considered consistent.

In Method 1, the reach length is fixed, hence, K is fixed (Equation 17-17) and C must be modified by the empirical relationship:

$$C^* = 1 - (1 - C)^{\left(\frac{\Delta t^* + .5\Delta t}{1.5\Delta t}\right)} \qquad \text{(Eq. 17-31)}$$

where C^* is the modified routing coefficient required for use with Δt^*, C is the coefficient determined from Figure 17-13 or computed by Equation 17-28, Δt^* is the desired routing interval, and Δt is the routing interval determined from Equation 17-27. After selecting Δt^* the coefficient C^* is found by using either Equation 17-31 or Figure 17-14 (ES-1025 rev.)

Method 2 assumes that C and the desired routing interval Δt^* are fixed and the routing is made for a reach length L^*. From Equation 17-27, the desired travel time is:

$$K^* = \frac{\Delta t^*}{C} \qquad \text{(Eq. 17-32)}$$

From Equation 17-17 the proper routing reach length to match C and Δt^* is then:

$$L^* = (3600)(V)(K^*) \qquad \text{(Eq. 17-33)}$$

If L^* is less than the given reach length, L, the inflow hydrograph is repetitively routed until the difference between the sum of the L^*'s and L becomes less than the next L^*. The last routing in the reach is a fractional routing using C^* computed by Equation 17-31. The Δt used in Equation 17-31 is the time interval for routing through the fractional length increment of L, L^{**}. (See Example 17-11 Method 2).

If L^* is greater than the given reach length, L, the inflow hydrograph is routed once using Method 1. Example 17-11 illustrates the use of Methods 1 and 2.

Variability of routing parameters; selection of velocity, V.
As shown by preceding relationships, the magnitudes of the routing parameters C and K (and therefore of Δt) depend on the magnitude of the velocity V. For steady flow in natural streams this velocity varies with stage but the variation is not the same for all seasons of a year or for all reaches of a stream, nor does the velocity consistently increase or decrease with stage. For unsteady flow, velocity varies not only with stage but also with the rate of change of the stream flow.

These facts would appear to require a change in routing parameters for each operational step in a routing. But exploratory routings with the Convex method show that constant parameters must be used to conserve mass, that is, to make total outflow equal total inflow. The necessity for the use of constant parameters is a characteristic of coefficient routing equations, including not only Equation 17-21 but also with the Muskingum routing equation (refs. 2 and 3) and the Storage-Indication equations. Therefore all of the examples in this part show a use of constant parameters. In practice the parameters need not be constant for all steps of a routing but the more often they are changed the more likely that the total outflow will not equal total inflow.

The average, dominant, and peak velocities of one inflow hydrograph will nearly always differ from the corresponding velocities of another hydrograph. Even though a single value of V is used to get the constant values of C, K, and Δt for a routing, this V will nearly always be different for different inflow hydrographs to a reach. Each inflow hydrograph will need its own routing parameters determined from its own selected velocity. There are various methods of selecting the velocity.

One method, useful when a computer is used, computes the velocity as the average of velocities for all given discharges of the inflow hydrograph ≥ 50 percent of the peak discharge.

A manual method with the same objective as the machine method will be used in this chapter to make manual routings comparable to machine routings. In this method the dominant velocity of the inflow hydrograph is used to determine the parameters to be used in the routing. If the inflow hydrograph has a single peak the velocity is for a discharge equal to 3/4 of the peak inflow rate. If the inflow hydrograph has two or more peaks the velocity is for the discharge with the largest value of Tq, where:

$$Tq = (3/4\text{-discharge}) \times (\text{duration of } 3/4\text{-discharge}) \qquad \text{(Eq. 17-34)}$$

The use of Equation 17-34 is illustrated in Example 17-8. Some additional remarks concerning the selected velocity are given in the paragraph preceding Example 17-7.

<u>Examples</u>.- The Convex method is generally used for routing hydrographs through stream reaches. It can also be used, without any change in procedure for routing mass curves through reaches. Examples of both uses will be given. The method can be used for routing through reservoirs but for this it is not as efficient as the mass-curve method of Example 17-1; therefore no examples of reservoir routing are given in this part. Examples are given showing various aspects of Convex routing.

Example 17-7 - Basic routine using assumed parameters.

Example 17-8 - Routing with parameters determined from reach data and with local inflow added at bottom of reach.

Example 17-9 - "Reverse Routing" or determining the inflow hydrograph for a given outflow hydrograph.

Example 17-10 - Routing of Mass Curve and method of getting the outflow hydrograph.

Example 17-11 - Routing any hydrograph through any reach. Method 1 and Method 2 are compared.

For the following examples it is assumed that stage-discharge and stage-end-area curves are available for the routing reach. These curves are used for determining the velocity, V, after the dominant discharge of the inflow hydrograph is obtained. In preliminary work such curves may not be available, in which case the velocity can be estimated during a field trip to the stream area, or a suitable velocity assumed, and the routing made as a tentative study; such routings need verification by routings based on reach data before making firm decisions about a project.

In the first example the values of C and Δt are assumed; therefore the reach length and K do not directly enter into the work:

Example 17-7.--Route the triangular inflow hydrograph of Figure 17-15 by the Convex computational method. Use assumed values of C = 0.4 and Δt = 0.3 hours. There is no local inflow into the reach.

1. <u>Prepare the operations table.</u>
Suitable headings and arrangement are shown for the first three columns in Table 17-14. The "remarks" column is used here to explain the steps; it is not needed in routine work.

2. <u>Tabulate the inflow rates at accumulated times, using intervals of Δt.</u>
The accumulated times at intervals of Δt = 0.3 hours are shown in column 1 of Table 17-14. The inflow rates at these times are taken from the inflow hydrograph of Figure 17-15 and shown in column 2.

3. <u>Prepare the working equation.</u>
Since C = 0.4 then (1 - C) = 0.6 and the working equation is $O_2 = (1 - C)\ O_1 + C\ I_1 = 0.6\ O_1 + 0.4\ I_1$. When inflow ceases the working equation is $O_2 = 0.6\ O_1$.

4. <u>Do the routing.</u>
Follow the steps shown in the remarks column of Table 17-14.

The computational work in step 4 can usually be done on most desk-calculators by using a system of making the two multiplications and the addition in one machine operation.

The outflow hydrograph of Table 17-14 is plotted on Figure 17-15. The circled points are the outflow discharges obtained in the routing. Discharges between the points are found by connecting the points with a smooth curve. Sometimes the routing points do not define the peak region

Table 17-14 Basic operations in the Convex routing method.

Time (hrs)	Inflow, I (cfs)	Outflow, O (cfs)	Remarks
(1)	(2)	(3)	
0	0	0	Given.
.3	800	0 1/	$O_2 = 0.6(0) + 0.4(0) =$ zero
.6	1600	320	$O_2 = 0.6(0) + 0.4(800) = 320$
.9	2400	832	$O_2 = 0.6(320) + 0.4(1600) = 832$
1.2	3200	1459	$O_2 = 0.6(832) + 0.4(2400) = 1459$
1.5	4000	2155	$O_2 = 0.6(1459) + 0.4(3200) = 2155$
1.8	3520	2893	$O_2 = 0.6(2155) + 0.4(4000) = 2893$
2.1	3040	3144	$O_2 = 0.6(2893) + 0.4(3520) = 3144$
2.4	2560	3102	$O_2 = 0.6(3144) + 0.4(3040) = 3102$
2.7	2080	2885	$O_2 = 0.6(3102) + 0.4(2560) = 2885$
3.0	1600	2563	$O_2 = 0.6(2885) + 0.4(2080) = 2563$
3.3	1120	2178	$O_2 = 0.6(2563) + 0.4(1600) = 2178$
3.6	640	1755	$O_2 = 0.6(2178) + 0.4(1120) = 1755$
3.9	160	1309	$O_2 = 0.6(1755) + 0.4(640) = 1309$
4.2	0 2/	849	$O_2 = 0.6(1309) + 0.4(160) = 849$
4.5	0	509	$O_2 = 0.6(849) = 509$ I_1 = zero.
4.8	0	305	$O_2 = 0.6(509) = 305$ " " "
5.2	0	183	$O_2 = 0.6(305) = 183$ " " "
5.5	0	110	$O_2 = 0.6(183) = 110$ " " "
etc.	etc.	etc.	etc.

1/ Outflow starts at $\Delta t = 0.3$ hrs.

2/ Inflow ceases at 4.0 hrs.

well enough; this usually happens when the routing interval is large. In such cases the peak is estimated by use of a smooth curve or the routing is repeated using smaller intervals (see Example 17-11 for use of Δt^*).

The recession curve or tail of the outflow hydrograph continues to infinity, the discharges getting smaller with every step but never becoming zero. This is a characteristic of most routing methods. In practice the recession curve is either arbitrarily brought to zero at some convenient low discharge or the routing is stopped at some low discharge as shown in Figure 17-15.

The next example is typical of the routine used in practice. Routing parameters are obtained from reach data and local inflow is added in the conventional manner. Local inflow is the (usually) small flow from the contributing area between the head and foot of a reach. Local inflow and the inflow into the head of the reach together make up the total flow from the drainage area above the foot of the reach. The local inflow is generally given as a hydrograph made with reference to the foot of the routing reach. When it is added to the routed outflow the sum is the total outflow hydrograph.

Example 17-8.--The inflow hydrograph in Figure 17-16 is to be routed through a reach having a low-flow channel length of 14,900 feet and a valley length of 12,400 feet. Stage-discharge and stage-end-area curves for the reach are available (not illustrated). A hydrograph of local inflow is given in Figure 17-16. Obtain the total outflow hydrograph for the reach.

1. Determine the discharge to be used for getting the velocity V.
The inflow hydrograph has two peaks and it is not readily apparent which peak is the dominant one, therefore the rule expressed by Equation 17-34 will be used. The 3/4-discharge for the first peak is 3,750 cfs with a duration of 2.63 hours; for the second, 2,680 cfs with a duration of 5.35 hours. Then Tq = 3750(2.63) = 9,850 cfs-hrs for the first peak and Tq = 2680(5.35) = 14,320 cfs-hrs for the second, therefore the second discharge will be used.

2. Determine the velocity, V.
Enter the stage-discharge curve for the reach with the selected 3/4-discharge from step 1 and find the stage for that flow. Then enter the stage-end-area curve with that stage and get the end-area in square feet. The velocity is the discharge divided by the end area. For this example V will be taken as 3.0 fps.

3. Determine K.
The reach has two lengths, one for the low-flow channel, the other for the valley. From an examination of the stage-discharge curve and the inflow hydrograph it is evident that most of the flow will exceed the capacity of the low-flow channel, therefore use the valley length. This is given as 12,400 feet. By Equation 17-17, using T_t = K, the value of K = 12400/3600(3.0) = 1.15 hours by a slide-rule computation.

4. Determine C.
Enter Figure 17-13 with V = 3.0 fps and find C = 0.65.

5. Compute Δt.
Using results from steps 3 and 4, and by Equation 17-27, Δt = 0.65 (1.15) = 0.745 hours. Round to 0.75 hours.

6. Prepare an operations table for the routing.
Suitable headings and arrangement are shown in Table 17-15.

7. Tabulate accumulated time at intervals of Δt and the discharges for inflow and local inflow at those times.
The times are given in column 1 of Table 17-15, inflows in column 2, and local inflows in column 4. Inflows and local inflows are taken from the given hydrographs, which are shown in Figure 17-16.

8. Prepare the working equation.
From step 4, C = 0.65 so that (1 - C) = 0.35. The working equation is $O_2 = 0.35\ O_1 + 0.65\ I_1$.

9. Do the routing.
Follow the routine used in Table 17-14 to get the outflows for column 3 of Table 17-15.

10. Get the total outflow hydrograph.
Add the local inflows of column 4, Table 17-15, to the routed outflows of column 3 to get the total outflows for column 5. This step completes the example. The total outflow hydrograph is shown in Figure 17-16.

Note in Figure 17-16 that the routed outflow peaks are not much smaller than the inflow peaks. The first routed outflow peak is 93.0 percent of its respective inflow peak, and the second 97.7 percent of its inflow peak. The reach has relatively small storage when compared with the inflow volumes; the first inflow peak has less volume associated with it than the second and it is reduced more than the second.

The next example illustrates a routine sometimes needed to get the upstream hydrograph when the downstream one is given. The working equation for this routine is a rearranged form of Equation 17-21:

$$I_1 = \frac{1}{C} O_2 - \frac{(1 - C)}{C} O_1 \qquad \text{(Eq. 17-35)}$$

Example 17-9.--Obtain the inflow hydrograph of a reach from the total outflow hydrograph by use of reverse routing. The total outflow hydrograph and local inflow are given in Table 17-16.

1. Determine the routing coefficient C and the routing interval Δt.
Follow the procedure of steps 1 through 5 of Example 17-8. For this example C = 0.44 and Δt = 0.5 hrs.

Table 17-15 Operations table for Example 17-8.

Time (hrs.)	Inflow (cfs)	Outflow (cfs)	Local Inflow (cfs)	Total Outflow (cfs)
(1)	(2)	(3)	(4)	(5)
0	0	0	0	0
.75	380	0 1/	110	110
1.50	1400	247	430	677
2.25	3000	996	830	1826
3.00	4450	2299	1000	3299
3.75	5000	3697	890	4587
4.50	4600	4544	650	5194
5.25	3750	4580	460	5040
6.00	2800	4040	320	4360
6.75	2100	3234	2200	3454
7.50	1600	2497	180	2677
8.25	1280	1914	170	2084
9.00	1150	1502	210	1712
9.75	1210	1273	310	1583
10.50	1480	1232	470	1702
11.25	1880	1393	650	2043
12.00	2360	1710	830	2540
12.75	2880	2132	950	3082
13.50	3250	2618	1000	3618
14.25	3500	3029	970	3999
15.00	3580	3335	880	4215
15.75	3480	3494	780	4274
16.50	3240	3485	650	4135
17.25	2930	3326	550	3876
18.00	2600	3069	470	3539
18.75	2280	2764	400	3164
19.50	1980	2449	330	2779
20.25	1730	2144	280	2424
21.00	1480	1875	230	2105
21.75	1280	1618	190	1808
27.50	1130	1398	150	1548
23.25	980	1224	120	1344
24.00	850	1065	100	1165
24.75	720	925	90	1015
25.50	620	792	80	872
26.25	530	680	70	750
27.00	450	582	60	642
27.75	400	496	50	546
28.50	350	434	40	474
29.25	310	353	30	383
30.00	270	325	20	345
etc.	etc.	etc.	etc.	etc.

1/ Outflow starts at Δt = 0.75 hrs.

2. Prepare the operations table for the routing.
Suitable headings and arrangements are shown in Table 17-16.

3. Tabulate accumulated time at intervals of Δt and the discharges for total outflow and local inflow at those times.
The times are given in column 1 of Table 17-16, total outflows in column 2, and local inflows in column 3. The total outflow (but not the local inflow) is shown in Figure 17-17.

4. Determine the outflows to be routed upstream.
A value in column 2, Table 17-16, minus the corresponding value in column 3 gives the outflow for column 4, which contains the outflows to be routed upstream.

5. Prepare the working equation.
C is given in step 1 as 0.44. By Equation 17-35, $I_1 = 2.27\ O_2 - 1.27\ O_1$.

6. Do the routing.
The routine is slightly different from that in Table 17-14. Using values from Table 17-16, the sequence is: for outflow time 0.5 hrs, $I_1 = 2.27(0) - 1.27(0) = 0$, which is recorded for inflow time zero; at outflow time 1.0 hrs, $I_1 = 2.27(163) - 1.27(0) = 370$, recorded for inflow time 0.5 hrs; for outflow 1.5, $I_1 = 2.27(478) - 1.27(163) = 878$, recorded for inflow time 1.0 hrs; and so on. The work is easily done by accumulative positive and negative multiplication on a desk calculator. The inflow hydrograph to time 7.5 hours is plotted on Figure 17-17.

It will sometimes happen in reverse routing that the working equation gives negative values for the inflow. This occurs when the total outflow hydrograph or the local inflow is in error.

The next example shows the downstream routing of a mass curve of inflow. The routine is the same as that for Example 17-7. The outflow hydrograph can be obtained from the mass outflow curve by a series of simple calculations; these outflows must be plotted at midpoints of time increments, not at end points.

Example 17-10.--Route the mass curve of inflow of Figure 17-18 by the Convex method. There is no local inflow.

1. Determine the routing coefficient C and the routing interval Δt.
Follow the procedure of steps 1 through 5 of Example 17-8. For this example C = 0.40 and Δt = 0.3 hrs.

2. Prepare the operations table for the routing.
Suitable headings and arrangement are shown in Table 17-17.

Table 17-16 Operations table for Example 17-9

Time (hrs)	Total Outflow (cfs)	Local Inflow (cfs)	Outflow to be routed (cfs)	Inflow (cfs)
(1)	(2)	(3)	(4)	(5)
0	0	0	0	0
.5	120	120	0 1/	370
1.0	310	147	163	878
1.5	680	202	478	1508
2.0	1250	318	932	2278
2.5	1850	325	1525	2978
3.0	2490	325	2165	3398
3.5	3030	322	2708	3648
4.0	3440	318	3122	3793
4.5	3700	280	3420	3899
5.0	3900	269	3631	3819
5.5	3940	226	3714	3539
6.0	3840	203	3637	2972
6.5	3500	156	3344	2370
7.0	3000	85	2915	1800
7.5	2485	61	2424	1300
8.0	1960	31	1929	etc.
etc.	etc.	etc.	etc.	

1/ Outflow starts at $\Delta t = 0.5$ hours.

3. Tabulate accumulated time at intervals of Δt and the mass inflows at those times.
The times are given in column 1 of Table 17-17 and mass inflows in column 2.

4. Prepare the working equation.
C is given in step 1 as 0.40. By Equation 17-21, $O_2 = 0.6\ O_1 + 0.4\ I_1$.

5. Do the routing.
The routine is exactly the same as that in Table 17-14. For example, at inflow time 2.7 hrs, O_2 is computed using inflow and outflow for the previous time or $O_2 = 0.6(3707) + 0.4(5952) = 4605$ cfs-hrs.

(Note: If only the mass outflow is needed the work stops with step 5. If the outflow hydrograph is also needed, the following steps are also taken.)

6. Compute increments of outflow.
These are the differences shown in column 4, Table 17-17.

7. Compute average rates of outflow.
Dividing the increment of outflow of column 4, Table 17-17, by the increment of time (in this case, 0.3 hrs) gives the average rate of outflow for the time increment. For example, between 1.8 and 2.1 hours in Table 17-17, the time increment is 0.3 hrs and the outflow increment is 906 cfs-hrs; then the average rate is 906/0.3 = 3,020 cfs. The average rates must be plotted as midpoints between the two accumulated times involved; for this case, 3020 cfs is plotted at a time of (1.8 + 2.1)/2 = 1.95 hours.

The mass inflow, mass outflow, and rate hydrograph are plotted in Figure 17-18.

The next example shows how to route any hydrograph through any reach length. Methods 1 and 2 are compared.

Example 17-11.--Route the inflow hydrograph of Figure 17-19 through a reach 30,000 feet long. Assume no local inflow.

Method 1

1. Determine desired routing time interval, Δt^*.
Following the rule expressed in Equation 17-30, Δt^* will be 0.4 hrs.

2. Determine routing coefficient, C, and routing interval Δt.
If a stage-discharge-velocity table for a typical section in the reach is used, the average velocity V is determined using the method from page 17-54, and C is computed using Equation 17-28. If a rating table is not used the C or V must be assumed; in this case, let C = 0.72. Rearranging Eq. 17-28 gives $V = 1.7C/(1-C) =$

Table 17-17 Operations table for Example 17-10.

Time (hrs.)	Mass Inflow (cfs-hrs)	Mass Outflow (cfs-hrs)	Increment of Outflow (cfs-hrs)	Outflow Rate (cfs)
(1)	(2)	(3)	(4)	(5)
0	0	0	0	0
.3	120	0 1/	48	160
.6	480	48	173	577
.9	1080	221	344	1146
1.2	1920	565	542	1806
1.5	3000	1107	757	2523
1.8	4128	1864	906	3020
2.1	5112	2770	937	3123
2.4	5952	3707	898	2993
2.7	6648	4605	817	2723
3.0	7200	5422	711	2370
3.3	7608	6133	590	1966
3.6	7872	6723	460	1533
4.2	7992	7183	324	1080
4.5	7992	7507	194	647
4.8	7992	7701	116	387
5.2	7992	7817	70	233
5.5	7992	7887	etc.	etc.
etc.	etc.	etc.		

1/ Outflow starts at Δt = 0.3 hours

$(1.7)(.72)/0.28 = 4.37$ fps. Combining Equations 17-17 and 17-27,
$K = \frac{L}{3600V} = \frac{\Delta t}{C}$ or $\Delta t = \frac{CL}{3600V} = (.72)(30000)/(3600)(4.37) =$

1.37 hrs. Use 1.4 hrs. Δt is also the wave travel time through the entire reach.

3. Determine C*
Using Equation 17-31 with $\Delta t = 1.4$ hrs, $\Delta t^* = 0.4$ hrs, and $C = 0.72$

$$C^* = 1 - (1-.72)^{\left(\frac{0.4+0.5(1.4)}{1.5(1.4)}\right)} = 1-(.28)^{\left(\frac{1.1}{2.1}\right)} = 1-(.28)^{0.524} =$$

$1-0.51 = 0.49$.

4. Prepare an operations table for the routing.
Suitable headings and arrangement are shown in Table 17-18.

5. Tabulate accumulated time intervals of Δt^* and the inflow discharges for those times.
The times are given in column 1 Table 17-18, the inflows taken from Figure 17-19 in column 2.

6. Prepare the working equation.
From step 3, $C^* = 0.49$. Using Equation 17-21 $O_2 = (1 - C^*)\ O_1 + C^*I_1$ or $O_2 = 0.51\ O_1 + 0.49\ I_1$. Solutions of this equation can easily be made by accumulative multiplication or a desk calculator.

7. Do the routing.
Follow the routine of Table 17-14. The outflows are shown in column 3 of Table 17-18.

8. Determine the times for the outflow.
Outflow begins at the end of the first Δt (not Δt^*) interval.

With $\Delta t = 1.4$ hrs, show this time in column 4 of Table 17-18 in the row where outflow begins. Get succeeding times by adding Δt^* intervals, 0.4 hours in this case, as shown in column 4. In plotting or otherwise displaying the inflow and outflow hydrographs they are put in their proper time order, using columns 1 and 4, as shown in figure 17-19.

Method 2

1. Determine desired routing time interval, Δt^*.
Same as Method 1, $\Delta t^* = 0.4$ hr.

2. Determine routing coefficient C.
The routing coefficient "C" for each subreach is computed from the outflow hydrograph of the preceding subreach as done in Step 2, Method 1. A constant C may be used for the entire reach but the resultant hydrograph will vary from one produced by recomputing C for each subreach. For simplicity in this example, a constant $C = 0.72$ is assumed. $V = 4.37$ fps.

Table 17-18 Operations table for Example 17-11 Method 1.

Time Inflow (hrs)	Inflow (cfs)	Outflow (cfs)	Time Outflow (hrs)
(1)	(2)	(3)	(4)
0	0	0	
.4	260	0 1/	1.4
.8	980	127	1.8
1.2	2100	545	2.2
1.6	3120	1307	2.6
2.0	3500	2195	3.0
2.4	3220	2834	3.4
2.8	2630	3023	3.8
3.2	1960	2830	4.2
3.6	1470	2404	4.6
4.0	1120	1946	5.0
4.4	840	1541	5.4
4.8	630	1198	5.8
5.2	455	920	6.2
5.6	345	692	6.6
6.0	265	522	7.0
6.4	180	396	7.4
6.8	130	290	7.8
7.2	100	212	8.2
7.6	75	157	8.6
8.0	60	117	9.0
8.4	45	89	9.4
8.8	35	67	9.8
9.2	20	51	10.2
9.6	10	36	10.6
10.0	0	23	11.0
etc.	etc.	12	11.4
		6	11.8
		3	12.2
		2	12.6
		1	13.0
		etc.	etc.

1/ Outflow starts at Δt = 1.4 hours

3. Determine length of subreach L^*.
This is the length of reach required to satisfy the relationship of Equation 17-26 with C = 0.72 and $\Delta t^* = 0.4$ hrs. Combining Equations 17-26 and 17-17 (let $K = T_t$) we have $L^* = (\Delta t)(V)(3600)/C = (0.4)(4.37)(3600)/0.72 = 8740$ ft.

4. Compare the total of subreach lengths, ΣL^* with the given reach length, L.
For $\Sigma L^* \leq L$ go to step 5
For $\Sigma L^* > L$ go to step 7

In this example $\Sigma L^*_{n=1} = 8740$

$\Sigma L^*_{n=2} = 17480$

$\Sigma L^*_{n=3} = 26220$

$\Sigma L^*_{n=4} = 34960$

Therefore, the first three routings are made by going to step 5 and the last routing by going to step 7.

5. Prepare working equation and do the routing.
Using Equation 17-21 and the routing coefficient computed in step 2, $O_2 = (1 - C)O_1 + CI_1 = 0.28\ O_1 + 0.72I_1$. The outflows for each subreach are shown in Table 17-19.

6. Go to step 2.

7. Determine the length of the remaining subreach to be routed.
Subtract the ΣL^* of the 3 completed routings, i.e., 26220 ft from the total reach length to get the remaining reach length to be routed. $L^{**} = 30000 - 26220 = 3780$ ft.

8. Determine the Δt time interval for the remaining subreach.
The time interval used here is the same as the wave travel time through the remaining subreach. Combining Equations 17-17 and 17-27 as in step 2 Method 1 $\Delta t^{**} = \frac{CL^{**}}{3600V} = (0.72)(3780)/(3600)(4.37) = 0.173$ hrs.

9. Determine the modified routing coefficient C^*.
Using Equation 17-31 with $\Delta t^{**} = 0.173$, $\Delta t^* = 0.4$ and C = 0.72,

$$C^* = 1-(1-C)^{\left(\frac{\Delta t^* 0.5\Delta t^{**}}{1.5\Delta t^{**}}\right)} = 1-(1-.72)^{\left(\frac{0.4+0.5(.173)}{1.5\ (.173)}\right)} =$$

$$1-(.28)^{\left(\frac{.4865}{.2595}\right)}\ 1-(.28)^{1.89} = 1 - .090 = .91$$

10. Prepare working equation.
Following Method 1 $O_2 = (1-C^*)O_1 + C^*I_1 = (1 - .91)\ O_1 + 0.91I_1 = .09\ O_1 + 0.91\ I_1$.

11. Do the routing.
The outflow for the fractional routing are shown in column 6 Table 17-19.

12. Determine the time for the routing.
The hydrograph for each subreach routing is set back one Δt time interval. In this example the first three routings are set back 0.4 hrs each and the last (fractional) routing is set back 0.173 hrs (round to 0.2 hrs). See column 7 Table 17-19 and Figure 17-19.

When C^* and Δt^* are used and local inflow is to be added, the local inflow must be used in its actual time position regardless of Δt and Δt^*. That is, the local inflow is not shifted back or forth because it is not affected by the use of C^* and Δt^*.

* *

Effects of transmission losses on routed flows
A flood hydrograph is altered by transmission losses occurring during passage of the flow through a reach. The amount of loss depends on the percolation rate of the channel, the wetted perimeter of channel during flow, and the duration of flow for a particular wetted perimeter (Chapter 19). Transmission loss varies with the amount of flow in the channel which means that the most accurate method of deducting the transmission loss from the routed flow will be on an incremental flow basis. It is seldom worthwhile to handle it in this manner unless the transmission loss is very large.

An acceptable practice for handling transmission losses is to route the inflow hydrograph in the usual manner and afterwards deduct a suitable quantity of flow from the outflow hydrograph (mainly from the rising limb). If that outflow is to be routed downstream again, the manner of flow deduction will not be critical. In some cases it may be reasonable to assume that local inflow will be completely absorbed by transmission losses, thus no local inflow is added to the unmodified outflow hydrograph. In other cases local rainfall may completely satisfy transmission losses, requiring unmodified local inflow to be added to the unmodified outflow hydrograph. The use of detailed procedures outlined in Chapter 19, "Transmission Losses", may be necessary for more complex situations.

Routing through a system of channels
The methods of channel routing given in Examples 17-7 through 17-11 are used for individual reaches of a stream. Ordinarily a routing progresses from reach through reach until the stages, rates, or amounts of flow are known for selected points in the entire stream system of a watershed. The method of progression will be illustrated using a schematic diagram or "tree graph" of a stream system. A typical graph is given in Figure 17-20. It does not need to be drawn to scale. The main purpose of the graph is to show the reaches in their proper relationship to each other, but various kinds of data can be written down at their respective points of application to make the graph a complete reference during the routing.

Routing through a stream system begins at the head of the uppermost reach. If there is more than one possible starting place, as in Figure 17-20, the most convenient should be chosen.

Table 17-19 Operation table for Example 17-11 Method 2

Time Inflow (hrs)	Inflow (cfs)	Outflow $\Sigma L^*=8740$ (cfs)	Outflow $\Sigma L^*=17480$ (cfs)	Outflow $\Sigma L^*=26220$ (cfs)	Outflow $\Sigma L^*=30000$ (cfs)	Outflow Time (hrs)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	0					
.4	260	0 1/				
.8	980	187	0			
1.2	2100	758	135	0		
1.6	3120	1724	584	97	0 2/	1.4
2.0	3500	2729	1405	447	88	1.8
2.4	3220	3284	2358	1137	415	2.2
2.8	2630	3238	3025	2016	1072	2.6
3.2	1960	2800	3178	2743	1931	3.0
3.6	1470	2195	2906	3056	2670	3.4
4.0	1120	1673	2394	2948	3021	3.8
4.4	840	1275	1875	2549	2955	4.2
4.8	630	962	1442	2064	2586	4.6
5.2	455	723	1096	1617	2111	5.0
5.6	345	530	827	1242	1661	5.4
6.0	265	397	613	944	1280	5.8
6.4	180	302	457	706	974	6.2
6.8	130	214	345	527	730	6.6
7.2	100	154	251	396	545	7.0
7.6	75	115	181	292	409	7.4
8.0	60	86	133	212	303	7.8
8.4	45	67	99	155	220	8.2
8.8	35	51	76	115	161	8.6
9.2	20	40	58	87	119	9.0
9.6	10	25	45	66	90	9.4
10.0	0	14	31	51	68	9.8
etc.	etc.	4	19	36	53	10.2
		1	8	24	38	10.6
		0	3	13	25	10.0
		etc.	1	6	14	11.4
			0	2	7	11.8
			etc.	1	2	12.2
				0	1	12.6
				etc.	0	13.0
					etc.	

1/ Outflow from subreach 1, 2, & 3 starts $\Delta t^* = 0.4$ hours after inflow starts into each subreach.

2/ Outflow from subreach 4 starts $\Delta t = 0.2$ hours (rounded from 0.17 hours) after inflow starts into subreach 4.

The first major step in routing through a stream system is to develop the routing parameters, C and Δt, for each reach. Many times it is necessary to use Δt^* to make the routing interval uniform through the stream system; these parameters should be obtained before the routing begins. The method of developing the parameters C, K, and Δt is given in steps 1 through 5 of Example 17-8. The method of determining C* and Δt^* is given in steps 1 through 3 of Example 17-11.

The second major step is the development of the inflow hydrographs at heads of uppermost reaches and of local inflow hydrographs for all reaches. The methods of Chapter 16 are used.

The third major step is the routing. For routing any particular flood on the stream system pictured in Figure 17-23 a suitable sequence is as follows:

1. Route the inflow hydrograph at (a) through reach (a,b).

2. Add local inflow of reach (a,b) to the routed outflow to get the total outflow hydrograph, which becomes the inflow hydrograph for reach (b,c). It should be noted here that local inflow for a reach is usually added at the foot of the reach. These may be circumstances, however, in which the local inflow should be added at the beginning of the reach. The proper sequence for adding local inflow can be determined only by evaluating each reach.

3. Route the total outflow from reach (a,b) through reach (b,c).

4. Add local inflow of reach (b,c) to the routed outflow to get the total outflow hydrograph for that tributary.

5. Route the inflow hydrograph at (d) through reach (d,c).

6. Add local inflow of reach (d,c) to the routed outflow to get the total outflow hydrograph at point (c).

7. Add the total outflow hydrographs from reaches (b,c) and (d,c), steps 4 and 6, to get the total outflow hydrograph at point (c).

8. Route the total hydrograph at point (c) through reach (c,f).

9. Add local inflow of reach (c,f) to the routed outflow to get the total outflow hydrograph at point (f).

10. Route the inflow hydrograph at point (e) through reach (e,f).

11. Add local inflow of reach (e,f) to the routed outflow to get the total outflow hydrograph for that tributary.

12. Route the inflow hydrograph at point (g) through reach (g,f).

13. Add local inflow of reach (g,f) to the routed outflow to get the total outflow for that tributary at point (f).

14. Add the total outflow hydrographs from reaches (c,f), (e,f), and (g,f), steps 9, 11, and 13 to get the total outflow hydrograph at point (f).

15. Route the total hydrograph at point (f) through reach (f,h).

16. Add local inflow of reach (f,h) to the routed outflow to get the total outflow hydrograph for reach (f,h).

17. Route the total hydrograph at point (h) through reach (h,i).

18. Add local inflow of reach (h,i) to the routed outflow to get the total outflow hydrograph for reach (h,i).

19. Route the inflow hydrograph at point (j) through reach (j,k).

20. Add local inflow of reach (j,k) to the routed outflow to get the total outflow hydrograph for reach (j,k).

21. Route the hydrograph at point (k) through reach (k,i).

22. Add local inflow of reach (k,i) to the routed outflow to get the total outflow hydrograph for this tributary.

23. Add the total outflow hydrographs from reaches (h,i) and (k,i), steps 18 and 22, to get the total outflow hydrograph for point (i).

24. Route the hydrograph at point (i) through reach (i,ℓ).

25. Add local inflow of reach (i,ℓ) to the routed outflow to get the total outflow hydrograph at point (ℓ). This completes the routing for a particular flood on this stream system.

When manual computations are used, an operations table with times, inflow hydrographs and local inflows tabulated in their proper sequence is useful. Blank columns are left for the routed outflows and total outflows, which are tabulated as routing progresses. Above the appropriate columns the required data and routing parameters are tabulated so that the table becomes a complete reference for the routing. A sample operations table for routing by Method 2 is shown as Table 17-20. After the inflow hydrograph and local inflows are tabulated the sequence of the work is as follows:

Tabulate the reach numbers in the order in which the routing will progress; perform the routings as shown in Example 17-11 and continue in this manner through the stream system. Note the routed outflow at 1.17 hrs which is rounded to 1.0 hrs. Theoretically, the outflow hydrograph should be interpolated on a multiple of Δt to properly position the hydrograph in relation to time. The linear interpolation equation is:

$$q_i = q_i + (q_{i+i} - q_i) \times \frac{\Delta t^* - \Delta t}{\Delta t^*} \qquad \text{(Eq. 17-36)}$$

where: q_i and q_{i+i} are consecutive discharges, Δt^* is the desired time interval and Δt is the required time interval of the partial routing. When using Method 2, Δt is always less than Δt^*.

If the interpolation step is omitted and the starting times rounded as in Table 17-20 it is recognized an error is introduced, the magnitude of which depends on the relative values of Δt and Δt^*.

*　　*

Table 17-20. Portion of a typical operations table for routing through a stream system.

REACH	16		15	14		
L(ft)	19300.		4000.	37000.		
V(fps)	2.9		4.8	3.2		
C	.63	.63	.74	.65	.65	
Δt*(hrs)	1.00	1.00	1.00	1.00	1.00	
L* (ft)	16541.	2759.	4000.	17486.	17486.	2028.
Δt (hrs)		.17	.17			.12
C*		.986	1.00			.998

Time	Inflow	Outflow Subrch1 @16540.'	Outflow	Local Inflow	Total Outflow	Outflow	Local Inflow	Total Outflow	Outflow Subrch1 @17486.'	Outflow Subrch2 @34972.'	Outflow	Local Inflow	Total Outflow
hrs.	(All Discharges in cfs)												
0	0			0	0	0[2]	0	0				80	80
1	450	0	0[1]	190	190	190	40	230	0			80	80
2	1680	284	280	700	980	980	140	1020	150	0	0[3]	80	80
3	3600	1153	1151	1000	2151	2151	300	2451	715	97	97	600	697
4	5340	2698	2678	810	3488	3488	445	3933	1844	498	498	850	1248
5	6000	4363	4340	520	4860	4860	500	5360	3202	1371	1371	690	2061
6	5520	5394	5380	315	5695	5695	460	6155	4605	2560	2560	410	2970
7	4500	5473	5472	205	5677	5677	380	6057	5612	3887	3887	200	4087
8	3360	4860	4868	120	4988	4988	280	5268	5901	5007	5007	100	5107
9	2520	3915	3928	75	4003	4003	210	4213	5490	5588	5588	80	5668
10	1920	3036	3048	45	3093	3093	160	3253	4660	5525	5525	80	5605
11	1440	2333	2343	25	2368	2368	120	2488	3745	4963	4963	80	5043
12	1080	1770	1778	15	1793	1793	90	1883	2928	4173	4173	80	4253
13	780	1335	1341	10	1351	1351	65	1416	2249	3365	3365	80	3445
14	580	986	990	8	998	998	50	1048	1707	2640	2640	80	2720
15	450	735	739	5	744	794	35	779	1279	2034	2037	80	2117
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

1/ Outflow begins at 1.00 hrs., rounded from 1.17 hrs.
2/ Outflow begins at 0.00 hrs., rounded from 0.17 hrs.
3/ Outflow begins at 2.00 hrs., rounded from 2.12 hrs.

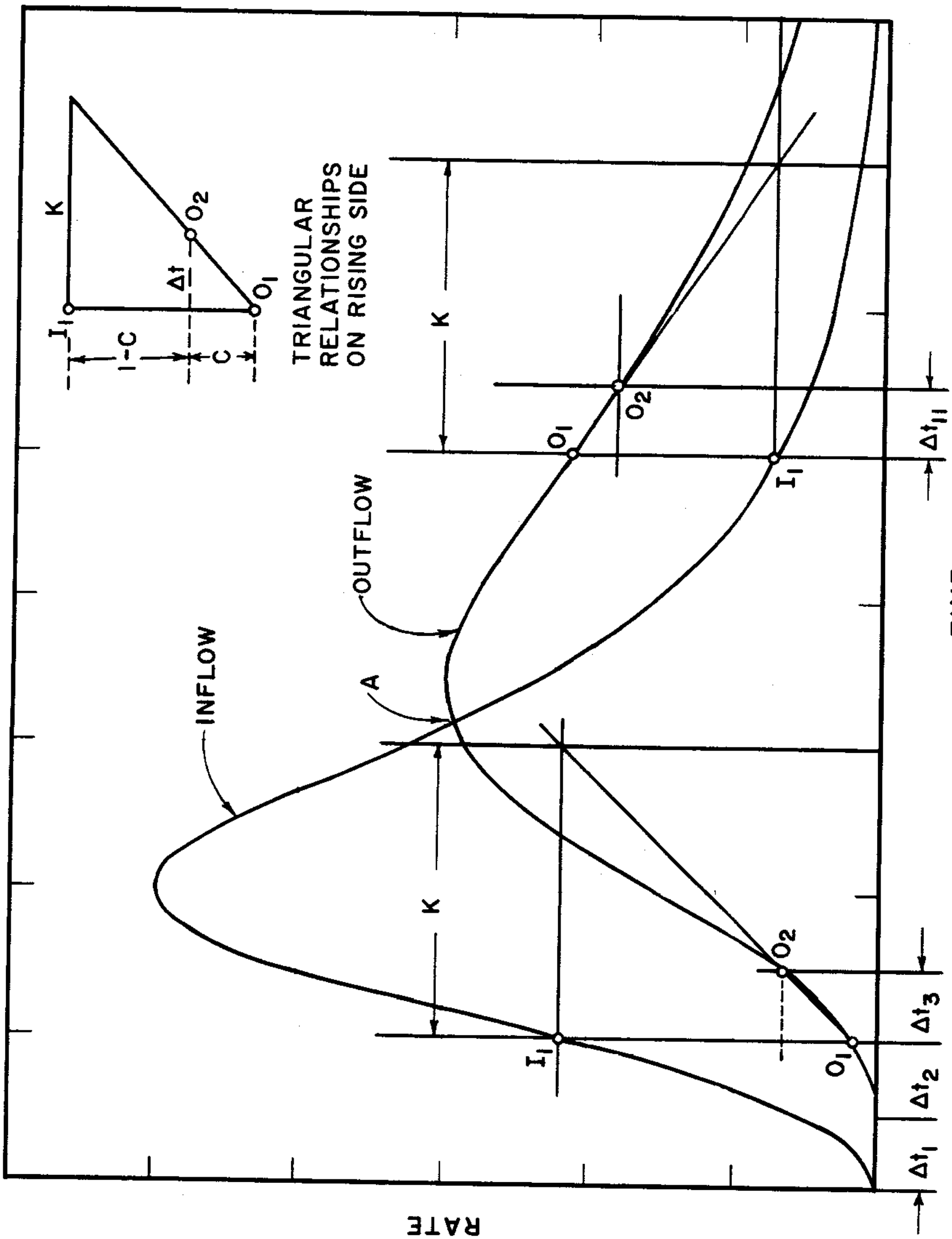


Figure 17-12. Relationships for Convex method of channel routing.

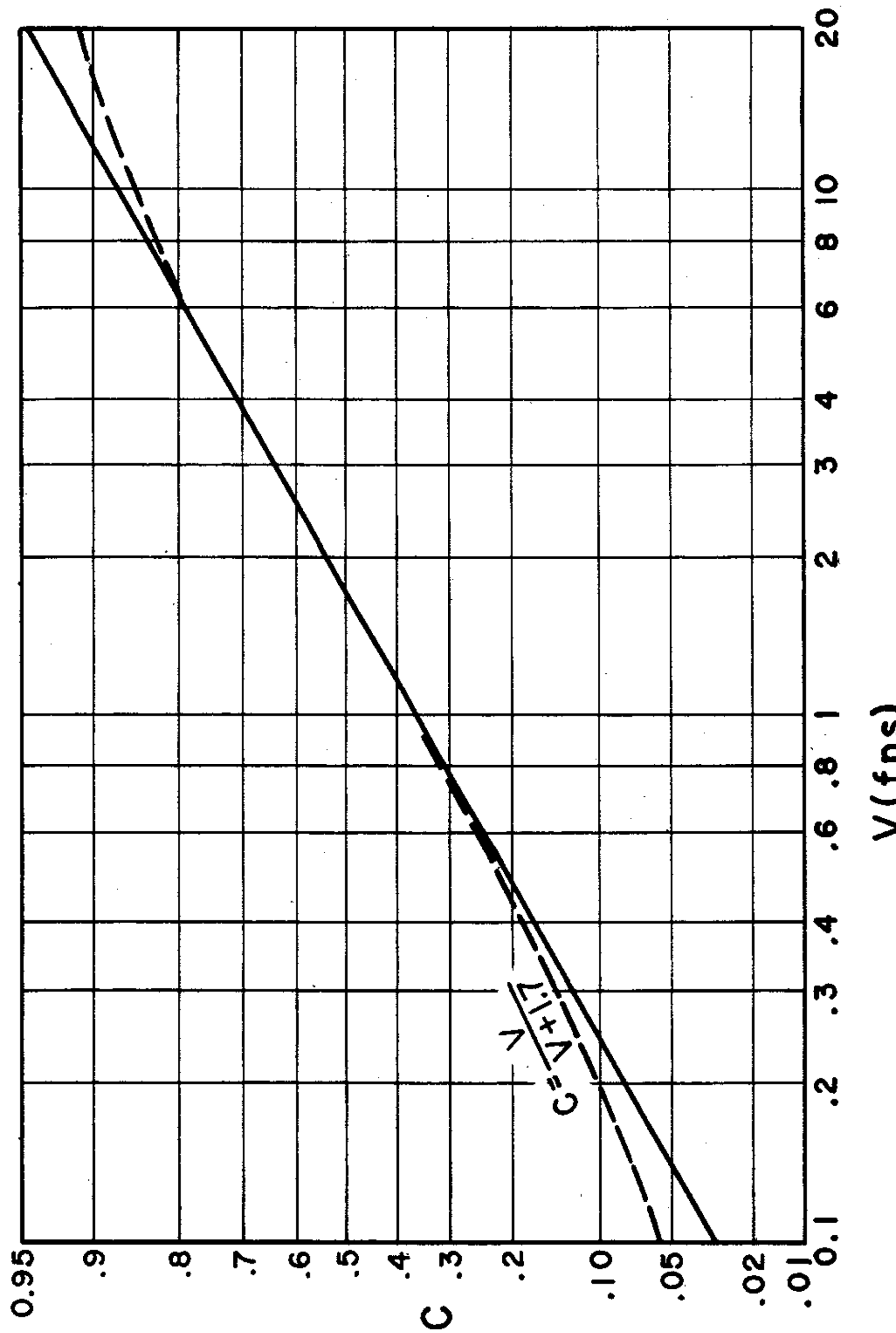
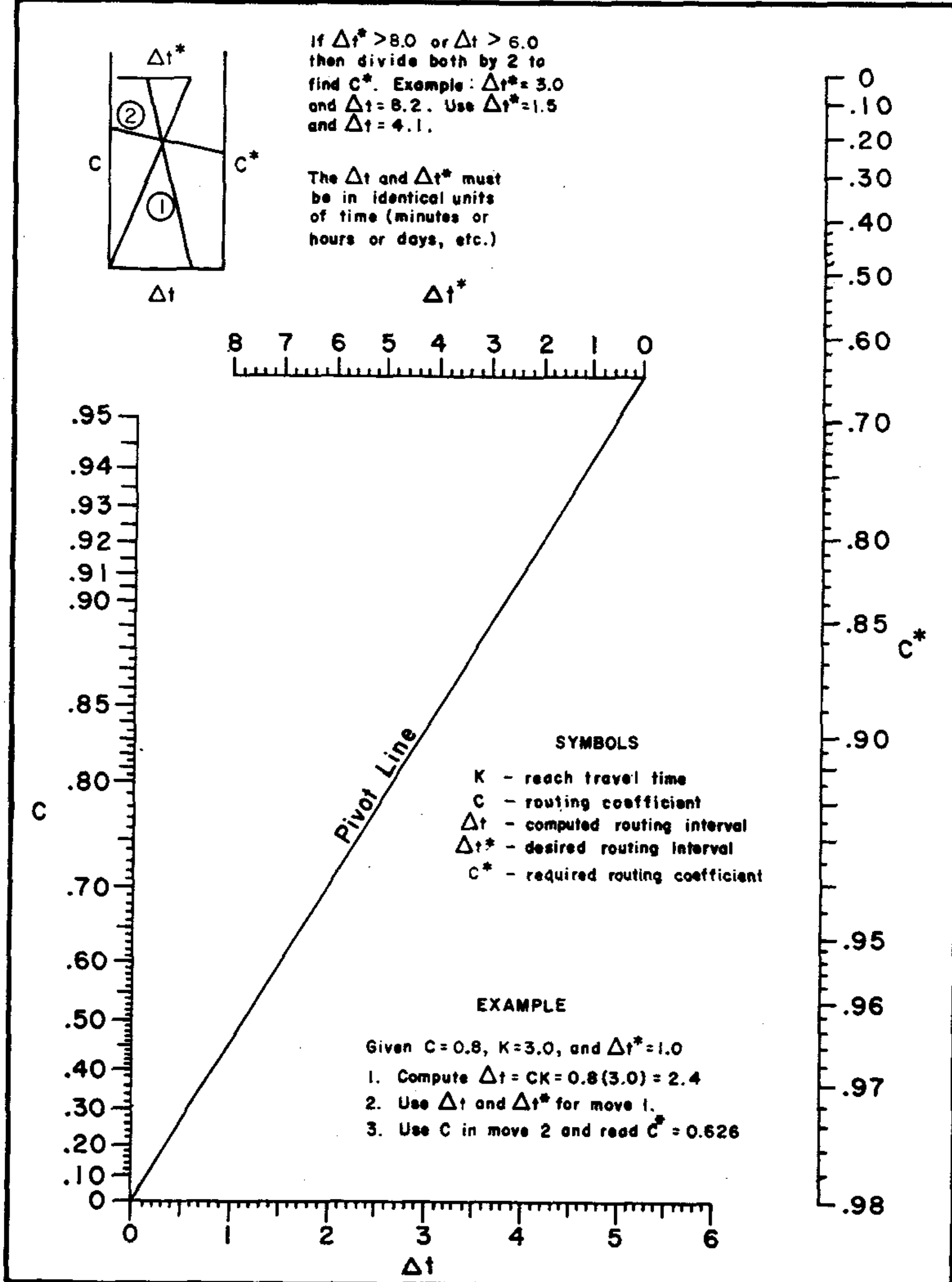


Figure 17-13. Convex routing coefficient versus velocity.

HYDROLOGY: NOMOGRAPH FOR SOLUTION OF $C^* = 1-(1-C)^{\frac{\Delta t^* + \Delta t}{1.5\Delta t}}$



REFERENCE
Equation by W. A. Styner
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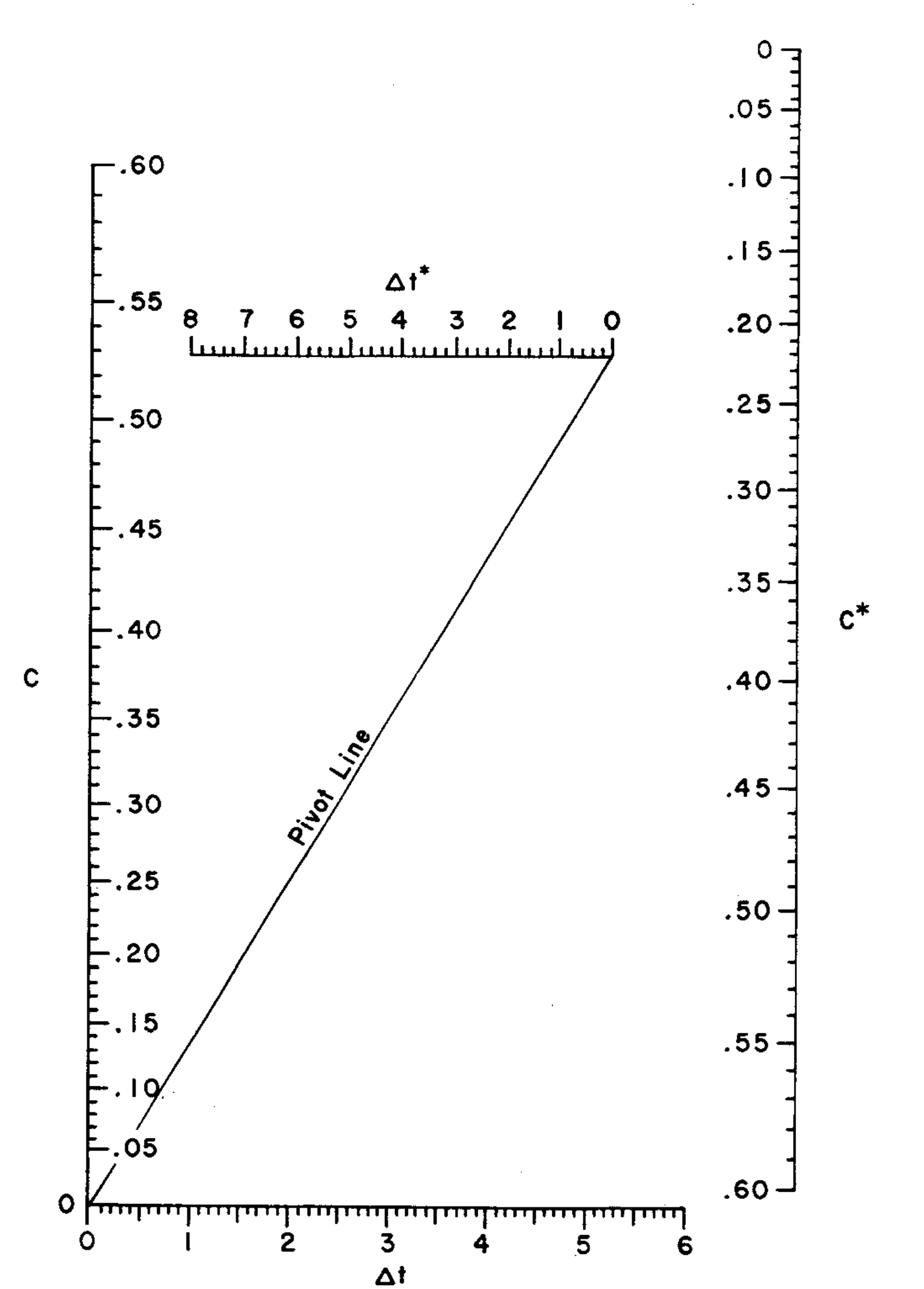
SHEET 1 OF 2
DATE 6-71

Figure 17-14. ES-1025 rev. sheet 1 of 2.

HYDROLOGY: NOMOGRAPH FOR SOLUTION OF $C^{*}=1-(1-C)^{\frac{\Delta t^{*}+\Delta t}{1.5\,\Delta t}}$



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Equation by W. A. Styner	SOIL CONSERVATION SERVICE	ES-1025 REV
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		DATE 6 - 71

Figure 17-14. ES-1025 rev. sheet 2 of 2.

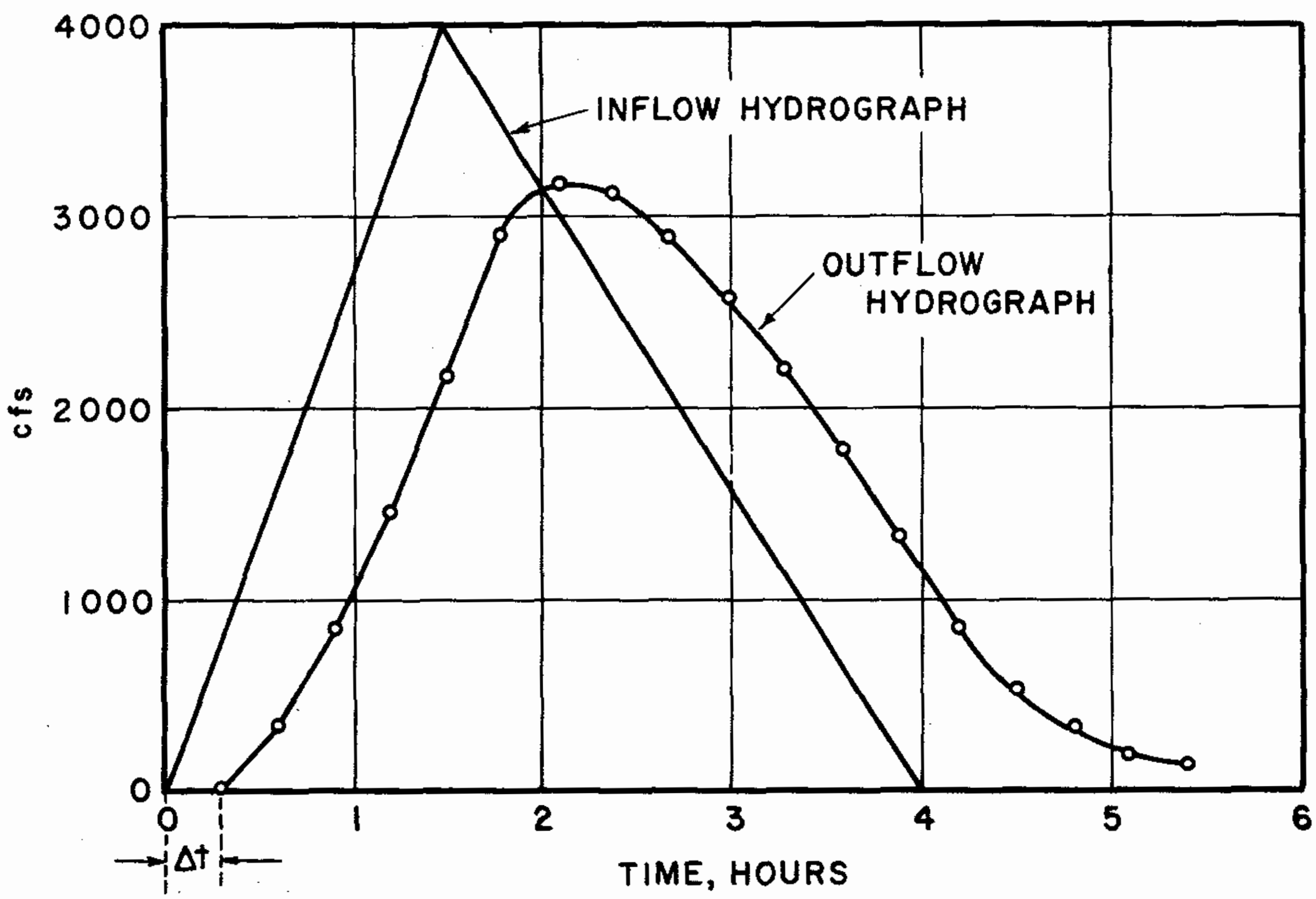

Figure 17-15. Inflow and routed outflow hydrograph for Example 17-7.

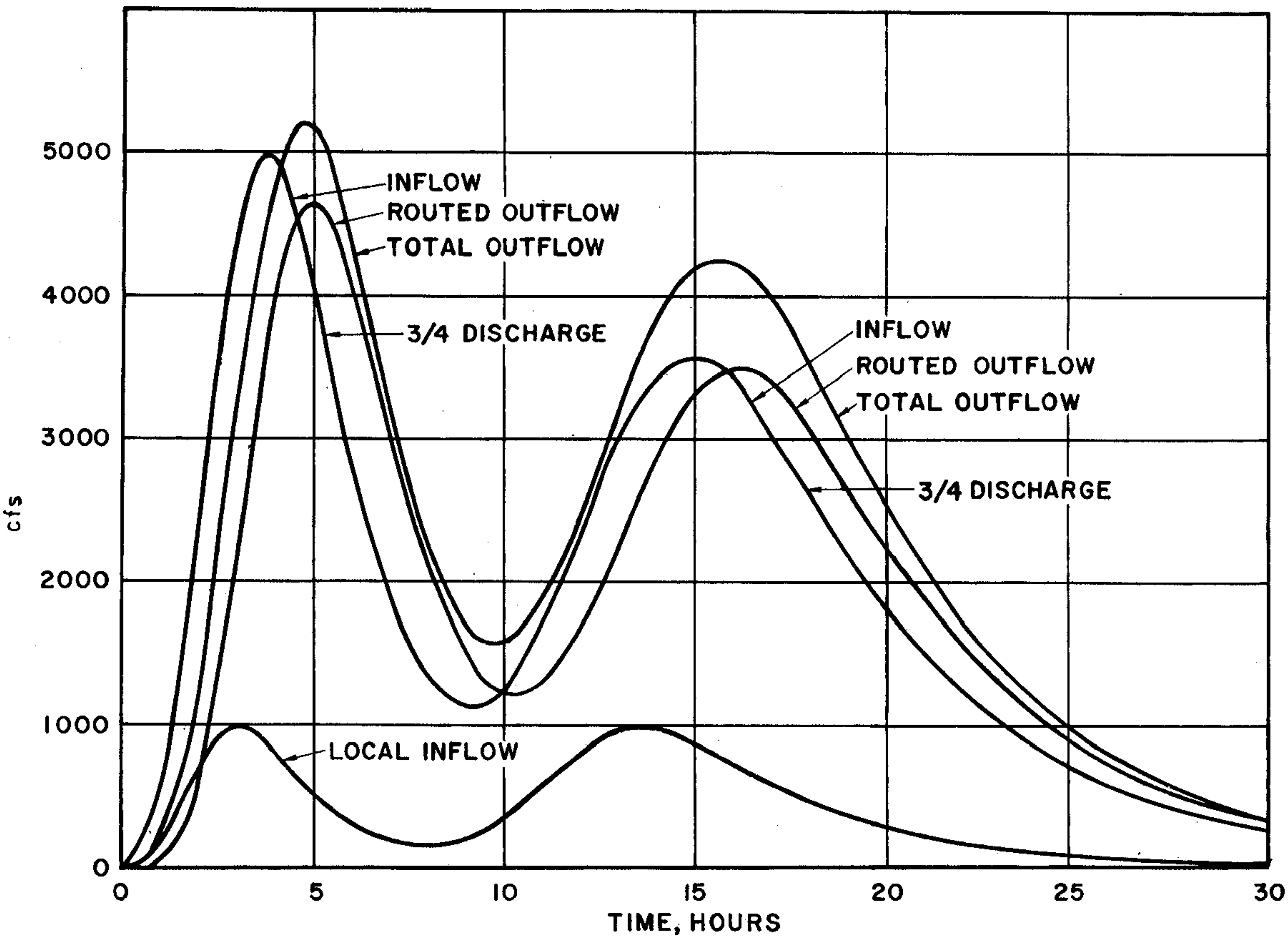

Figure 17-16. Inflow and routed outflow hydrograph for Example 17-8.

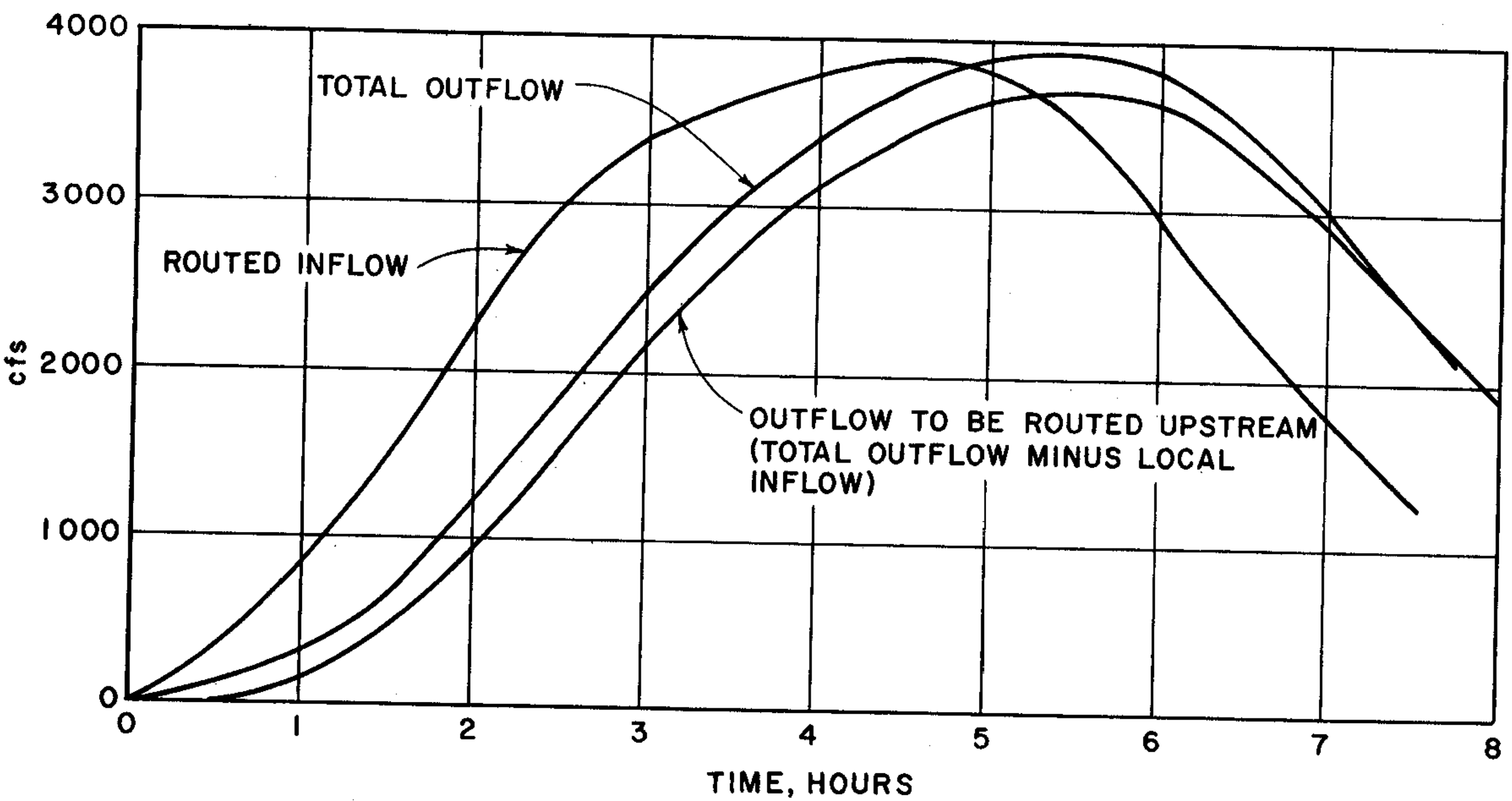

Figure 17-17. Outflow and routed inflow hydrograph for Example 17-9.

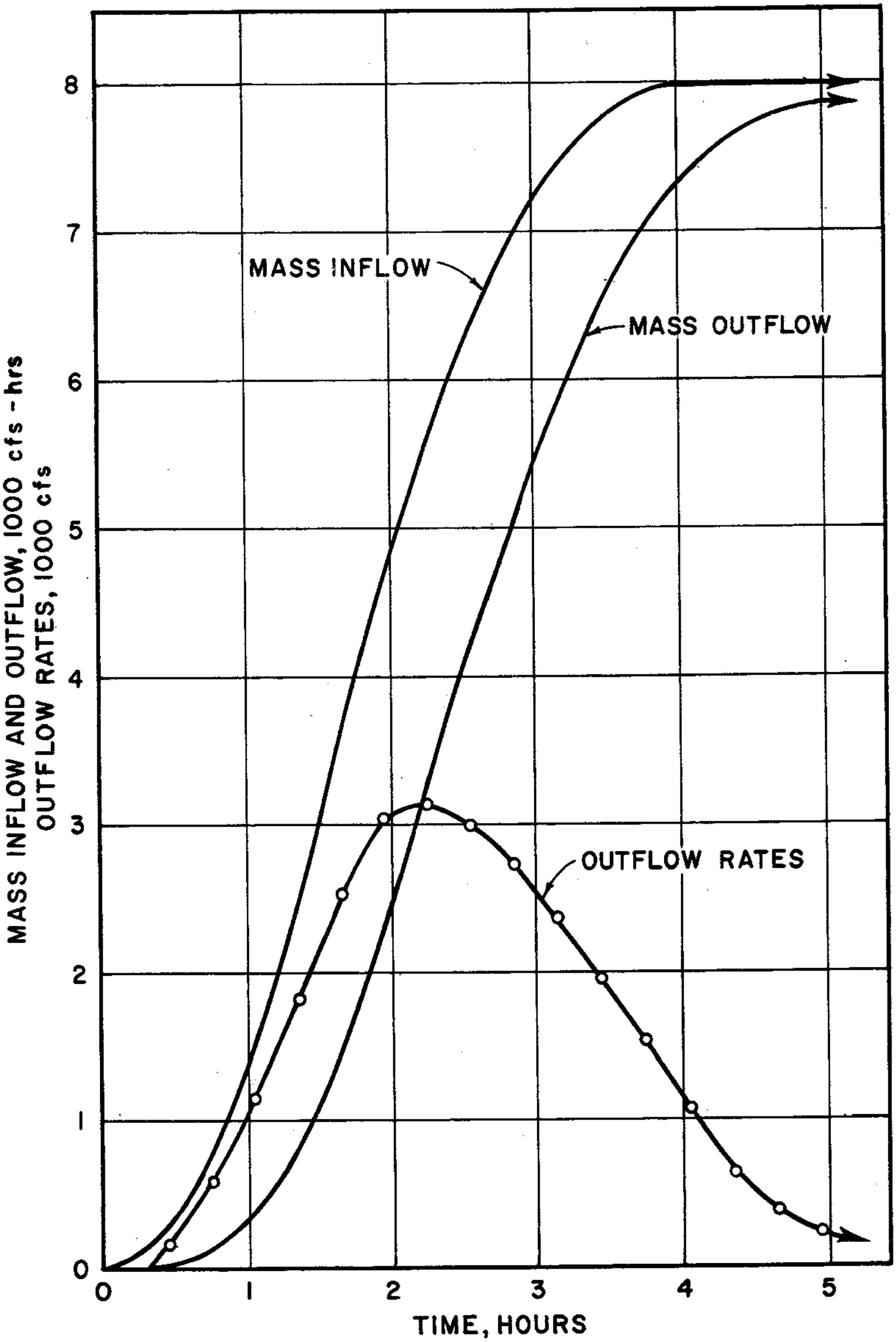

Figure 17-18. Mass inflow, mass outflow and rate hydrograph for Example 17-10.

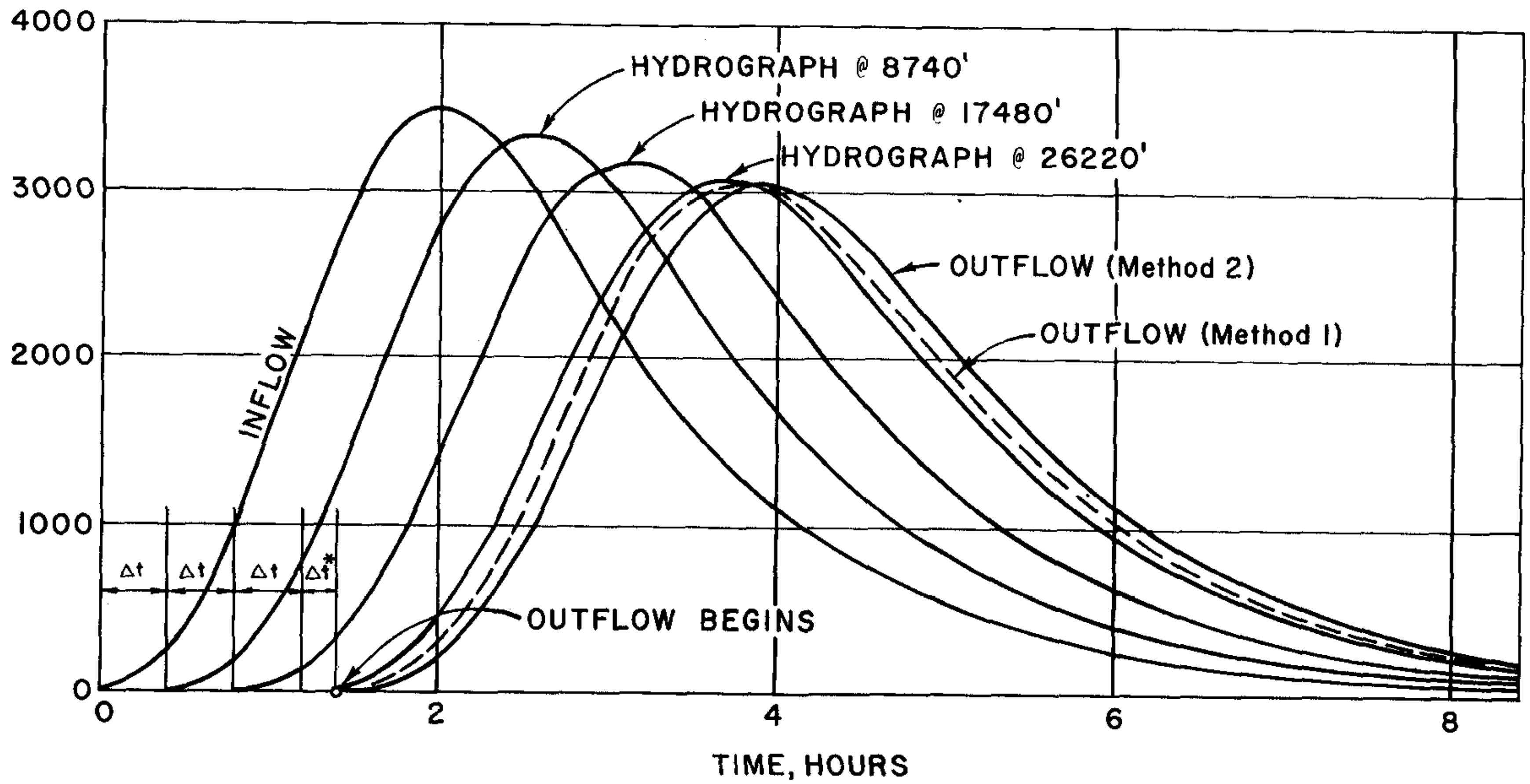

Figure 17-19. Inflow hydrograph and routed outflow hydrographs for Example 17-11, Method 1 and 2.

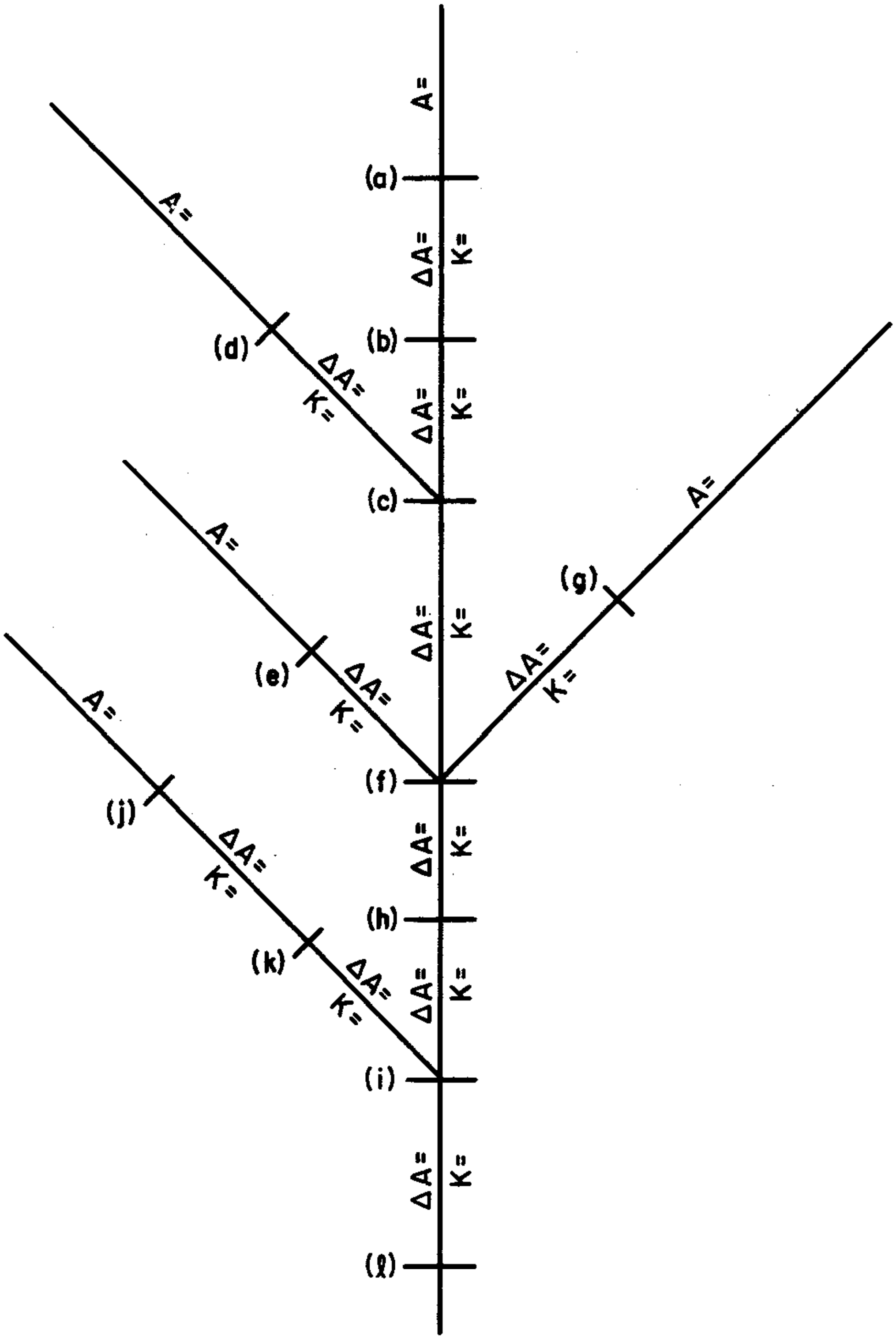

Figure 17-20. Typical Schematic diagram for routing through a system of channels.

Unit-Hydrograph Routing Methods

Principles of the unit hydrograph theory are given in Chapter 16. They apply to single-peaked hydrographs originating from uniform runoff on the contributing area but they can be extended to apply to more complex runoff conditions. Despite the limitations of the theory it has features that can be used in determining peak rates in stream reaches not only when the watershed is in a "present" unreservoired condition but also when it is controlled by many reservoirs. It is the ease with which complex systems of control structures are evaluated that has made the unit-hydrograph type of routing a popular method for many years. If suitable data are used the results are usually as good as those obtained by more detailed methods of routing.

In this part of the chapter the basic equations for unit-hydrograph routing will be given and discussed and some of their uses explained by means of examples. The unit-hydrograph method of routing gives only the peak rates of runoff. The peak-producing hydrograph, if it is needed, must be obtained in some other way.

Basic Equations

All of the unit-hydrograph working equations are derived from the relationship for the peak rate of a unit hydrograph:

$$q_p = \frac{K\ A\ Q}{T_p} \qquad \text{(Eq. 17-37)}$$

where q_p = peak rate in cfs

K = a constant (not the routing parameter used in the Convex method)

A = drainage area contributing runoff; in square miles

Q = average depth of runoff, in inches, from the contributing area

T_p = time to peak, in hours

By letting q_p, K, A, Q, and T_p stand for a watershed in one condition and using primed symbols q_p', K', A', Q', and T_p' for the same watershed in a condition being studied, then by use of Equation 17-37 it is evident that:

$$q_p' = q_p \frac{A'\ Q'\ T_p}{A\ \ Q\ \ T_p'} \qquad \text{(Eq. 17-38)}$$

which is a typical working equation of the unit hydrograph method. It can be used, for example, in determining the peak rates after establishment of land use and treatment measures on a watershed. In such work the present peaks, areas, runoff amounts, and peak times are known and it is only a matter of finding the change in runoff by use of Chapter 10 methods. The areas and peak times are assumed to remain constant.

When a floodwater retarding structure, or other structure controlling a part of the watershed, is being used in the "future" condition then the value of A' is reduced. And if there are releases from the structure then they must also be taken into account. For a project having structures controlling a total of A* square miles and having an average release rate of q* csm, the peak rate equation becomes:

$$q_p' = q_p \frac{(A' - A^*)}{A} \frac{Q'}{Q} \frac{T_p}{T_p'} + q^* (A^*) \qquad \text{(Eq. 17-39)}$$

When using Equation 17-39 to find the reduced peak rate the major assumption is that the structures are about uniformly distributed over the watershed. Another assumption is that all structures contribute to q*, but this is sometimes too conservative an assumption (see the section titled "Use of Equation 17-43 on large watersheds").

When $A' = A$ and $T_p' = T_p$, which is the usual case when evaluating land use and treatment effects, Equation 17-38 becomes:

$$q_p' = q_p \frac{Q'}{Q} \qquad \text{(Eq. 17-40)}$$

which is one of the basic expressions of the unit hydrograph theory. If the same simplification applies when evaluating structures then Equation 17-39 becomes:

$$q_p' = q_p \frac{A - A^*}{A} + q^* (A^*) \qquad \text{(Eq. 17-41)}$$

Equation 17-41 can be further simplified by using:

$$r = \frac{A^*}{A} \qquad \text{(Eq. 17-42)}$$

where r is the fraction of drainage area under control or the percent of control divided by 100. Using Equation 17-42 in Equation 17-41 gives:

$$q_p' = q_p (1 - r) + q^*(A^*) \qquad \text{(Eq. 17-43)}$$

Effects of storm duration and time of concentration

When the effects of a change in either the storm duration or the time of concentration must be taken into account, one way to do it is to use the following relation from Chapter 16:

$$T_p = a(D) + b(T_c) \qquad \text{(Eq. 17-44)}$$

where T_p = time to peak, in hours

a = a constant

D = storm duration, in hours, during which runoff is generated; it is usually less than the total storm duration.

b = a constant

T_c = time of concentration, in hours

As shown in Chapter 16, the constants a and b can be taken as 0.5 and 0.6 respectively, for most problems, in which case Equation 17-44 becomes:

$$T_p = 0.5\ D + 0.6\ T_c \qquad \text{(Eq. 17-45)}$$

Using Equation 17-45 in equations 17-37, 17-38, and 17-39 produces working equations in which either the storm duration or the time of concentration can be changed and the effect of the change determined. Such equations are not often used because the main comparison is usually between present and future conditions in which only runoff amount and drainage area will change. In special problems where storm duration must be taken into account there are other approaches that are more applicable (see the section titled "Use of Equation 17-43 on large watersheds").

Elimination of T_p

In many physiographic areas there is a consistent relation between T_p and A because there is a typical storm condition or pattern. The relationship is usually expressed as:

$$T_p = c\ A^d \qquad \text{(Eq. 17-46)}$$

where c is a constant multiplier and d is a constant exponent. Substituting cA^d for T_p in Equation 17-37 gives:

$$q_p = k\ A^{(1-d)}\ Q \qquad \text{(Eq. 17-47)}$$

where k = K/c. Letting (1 - d) = h, Equation 17-47 becomes:

$$q_p = k\ A^h\ Q \qquad \text{(Eq. 17-48)}$$

which is the working equation in practice. The parameters k and h are obtained from data for a large storm over the watershed or region being studied. Values of q_p at several locations are obtained either from streamflow stations or by means of slope-area measurements (Chapter 14): values of Q associated with each q_p are obtained from the station data or by use of rainfall and watershed data and methods of chapter 10; and drainage areas at each location are determined. A plotting of q_p/Q against A is made on log paper and a line of best fit is drawn through the plotting. The multiplier k is the intercept of the line where A = 1 square mile and the exponent h is the slope of the line. See the section titled "Use of Equations 17-48, 17-50, and 17-52" for an application of this procedure.

After h is known, the equivalent of Equation 17-38 is:

$$q_p' = q_p \left(\frac{A}{A}\right)^h \frac{Q'}{Q} \qquad \text{(Eq. 17-49)}$$

The k's cancel out in making this change.

In the "Concordant Flow" method of peak determination, Equation 17-48 is modified to take into account the effects of control structures and their release rates, with the working equation being:

$$q_p' = k\ A^h\ Q\ (1 - r) + q^*(A^*) \qquad \text{(Eq. 17-50)}$$

or:

$$q_p' = q_p\ (1 - r) + q^*(A^*) \qquad \text{(Eq. 17-51)}$$

which is the same as Equation 17-43 in form but where q_p is now determined from Equation 17-48.

Equations 17-39, 17-41, 17-43, 17-50, and 17-51 should be used only when the storm runoff volume does not exceed the storage capacity of the structure with the smallest capacity. If the runoff does exceed that capacity these equations must be modified further. Equation 17-50, for example, becomes:

$$q_p' = k\ A^h\ (Q - r\ Qs) + q^*(A^*) \qquad \text{(Eq. 17-52)}$$

where Qs is the average storage capacity of the structures. It is shown in Example 17-20 how Equation 17-51 and similar equations can be used even when the capacity varies from structure to structure.

Working equations for special cases

Additional equations can be developed from those given if a special problem arises in watershed evaluation. For an example, suppose that Equation 17-43 is to be used for determining the effects of a proposed system of floodwater retarding structures in a watershed, and that the evaluation reaches are so long that the percent of area reservoired varies significantly from the head to the foot of the reach. To modify Equation 17-43 for this case, let A* be the area reservoired, A the total area, and r = A*/A for the head of the reach; and let B* be the total area reservoired (including A*), B the total area (including A), and r" = B*/B for the foot of the reach. For evaluations to be made at the foot of the reach, Equation 17-43 then becomes:

$$q_p' = q_p \left(\frac{2 - r - r''}{2}\right) + q^* \left(\frac{A^* + B^*}{2}\right) \qquad \text{(Eq. 17-53)}$$

After first computing (2 - r - r")/2 = C' and (A* + B*)/2 = C" for the reach, the working equation becomes:

$$q_p' = q_p\ C' + q^*\ C'' \qquad \text{(Eq. 17-54)}$$

where C' and C" are the computed coefficients. Each evaluation reach requires its own set of coefficients.

Examples

The problems in the following examples range from the very simple to the complex, the latter being given to show that unit-hydrograph methods have wide application. For some complex problems, however, it will generally be more efficient to use the SCS electronic-computer evaluation program.

Use of Equation 17-40. - This basic expression of the unit hydrograph theory has many uses. The major limitation in its use is that Q and Q' must be about uniformly distributed over the watershed being studied. The following is a typical but simple problem.

Example 17-12.--A watershed has a peak discharge of 46,300 cfs from a storm that produced 2.54 inches of runoff. What would the peak rate have been for a runoff of 1.68 inches?

1. Apply Equation 17-40.
For this problem q_p = 46,300 cfs, Q = 2.54 inches, and Q' = 1.68 inches. By Equation 17-40 q_p' = 46300(1.68/2.54) = 30,604 cfs, which is rounded to 30,600 cfs.

Use of Equation 17-43 - The major limitations in the use of this equation are that both the runoffs and the structures must be about uniformly distributed over the watershed and that the stream travel times for the "future" condition must be about the same as for the "present." The following is a typical but simple problem.

Example 17-13.--A watershed of 183 square miles has a flood peak of 37,800 cfs. If 42 square miles of this watershed were controlled by floodwater retarding structures having an average release rate of 15 csm, what would the reduced peak be?

1. Compute r.
By Equation 17-42 r = 42/183 = 0.230 because A* = 42 and A = 183 square miles.

2. Apply Equation 17-43.
For this problem, q_p = 37,800 cfs, r = 0.230 from step 1, q* = 15 csm, and A* = 42 square miles. By Equation 17-43 q_p' = 37800(1 - 0.230) + 15(42) = 29,736 cfs, which is rounded to 29,700 cfs. This is the reduced peak.

Use of Equation 17-43 on large watersheds.- If Equation 17-43 is used for evaluating the effects of structures in a large watershed or river basin the releases from structures far upstream may not add to the peak rates in the lower reaches of the main stem. And if releases from certain upstream structures do not affect peaks far downstream then those structures also are not reducing the peak rates, therefore their drainage areas should not be used in the equation.

In problems of this kind the approach to be taken is relatively simple though there are supplementary computations to be made before the equation is used. The key step in the approach is finding the T_p for an evaluation flood and using only those areas and structures close enough to the sub-basin outlet to affect the peak rate of that flood. How this is done will be illustrated using the data and computations of Table 17-21. The data are for a sub-basin of 620 square miles, with a time of concentration of 48 hours. Storm durations for the floods to be evaluated will vary from 1 to over 72 hours, which means that the sub-basin T_p will also vary considerably.

Table 17-21 is developed as follows:

Column 1 lists the travel times on the sub-basin main stem from the outlet point to selected points upstream, which are mainly junctions with major tributaries. The first entry is for the outlet point.

Column 2 gives the total drainage area above each selected point.

Column 3 gives the increments of area.

Column 4 gives the accumulated areas, going upstream. These are the contributing areas when the flood's T_p is within the limits shown in column 1. For example, when T_p is between 3.5 and 9.1 hours, the contributing drainage area is 74 square miles. T_p must be at least 48 hours before the entire watershed contributes to the peak rate.

Column 5 shows the total areas controlled by structures.

Column 6 gives the increments of controlled area.

Column 7 gives the accumulated controlled areas, going upstream.

Column 8 gives values of r, which are computed using entries of columns 7 and 4.

Column 9 gives values of (1 - r), which are computed using entries of column 8.

Column 10 gives the total average release rate in cfs for the controlled areas of column 7. For this table the average release rate q* is 7 csm. Therefore the q*(A*) entry for a particular row is the column 7 area of that row multiplied by the average rate in csm.

Only columns 1, 9, and 10 are used in the remaining work. To determine the effect of the structures the q_p and T_p of the evaluation flood must be known, the proper entries taken from the table, and Equation 17-43 applied. For example, if q_p = 87,000 cfs and T_p = 24 hours for a particular flood, first enter column 1 with T_p = 24 hours and find the row to be used, in this case it is between T_t values of 21.1 and 28.0 hours; next select (1 - r) = 0.459 from column 9 of that row and q*(A*) = 1,491 cfs from column 10; finally, use Equation 17-43 which gives q_p' = 87000 (0.459) + 1491 = 41,424 cfs, which is rounded to 41,400 cfs.

Table 17-21 Data and working table for use of Equation 17-43 on a large watershed

T_t (hrs)	A (sq.mi.)	ΔA (sq.mi.)	Au (sq.mi.)	A* (sq.mi.)	ΔA^* (sq.mi.)	A^*_u (sq.mi.)	r	(1 - r)	$q^*(A^*)$ 1/ (cfs)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	620			359					
		8	8		0	0	0	1.000	0
2.0	612			359					
		6	14		3	3	.214	.786	21
3.5	606			356					
		60	74		24	27	.365	.635	189
9.1	546			332					
		90	164		43	70	.426	.574	490
15.3	456			289					
		80	244		56	126	.517	.483	882
21.1	376			233					
		150	394		87	213	.541	.459	1491
28.0	226			146					
		101	495		77	290	.586	.414	2030
31.0	125			69					
		98	593		48	338	.570	.430	2366
42.0	27			21					
		27	620		21	359	.580	.420	2513
48.0	0			0					

1/ Using an average rate of q* = 7 csm.

If any other point in the sub-basin is also to be used for evaluation of structure effects then a separate table is needed for that point.

Use of Equations 17-48, 17-50, and 17-52.- When streamflow data or slope-area measurements and Q estimates are available for a watershed and its vicinity, the information can be used to construct a graph of q_p/Q and A as shown in Figure 17-21. This is the graphical form of Equation 17-48. If a line with an intercept of 484 cfs/in. and slope of 0.4 can be reasonably well fitted to the data, as in this case, it means that the hydrograph shapes of these watersheds closely resemble the shape of the unit hydrograph of Figure 16-1 (see Chapter 16). Usually the slope will be 0.4 for other shapes of hydrographs (the reason for this is discussed in Chapter 15) but the intercept will vary. For the line of Figure 17-21, Equation 17-48 can be written:

$$q_p = 484 A^{0.4} Q \qquad \text{(Eq. 17-55)}$$

The following examples show some typical uses of the graph or its equation.

Example 17-14.--For a watershed in the region to which Figure 17-21 applies, A = 234 square miles and Q = 3.15 inches for a storm event. What is q_p?

1. Find q_p/Q for the given A.

Enter the graph with A = 234 square miles and at the line of relation find q_p/Q = 4,290 cfs/in.

2. Compute q_p.

Multiplying q_p/Q by Q gives q_p, therefore, q_p = 3.15(4290) = 13,500 cfs by a slide-rule computation.

If part of a watershed is controlled by floodwater retarding structures the graph can be used together with equation 17-50, as follows:

Example 17-15.--A watershed of 234 square miles has a system of floodwater retarding structures on it controlling a total of 103 square miles. Each structure has a storage capacity of 4.5 inches before discharge begins through the emergency spillway. Each structure has an average release rate of 15 csm. When the storm runoff Q is 4.1 inches what is the peak rate with (a) structures not in place, and (b) structure in place?

1. Determine the flood peak for the watershed with structures not in place.

Use the method of Example 17-14. Enter Figure 17-21 with A = 234 square miles and find q_p/Q = 4,290 cfs/in. Multiplying that result by Q = 4.1 inches gives q_p = 4.1(4290) = 17,600 cfs by a slide-rule computation. This discharge is ($k A^h Q$) in Equation 17-50.

2. Determine (1 - r).
From Equation 17-42 $r = A^*/A = 103/234 = 0.440$. Then $(1 - r) = 1 - 0.440 = 0.560$.

3. Determine the flood peak for the watershed with structures in place.
Use Equation 17-50 with the results of steps 1 and 2 and the given data for controlled area and release rate: $q_p' = 17600(0.560) + 15(103) = 11,410$ cfs, using a slide-rule for the multiplications. Round the discharge to 11,400 cfs.

If the storm runoff exceeds the storage capacities of the structures but the capacities are the same for all structures then Equation 17-52 can be applied as shown in the following example.

Example 17-16.--For the same watershed and structures used in Example 17-18 find the peak rates without and with structures in place when the storm runoff is 6.21 inches.

1. Determine the flood peak for the watershed with structures not in place.
Use the method of Example 17-14. Enter Figure 17-21 with A = 234 square miles and find $q_p/Q = 4,290$ cfs/in. This is $(k\ A^h)$ in Equation 17-52. Multiplying that result by Q = 6.21 inches gives $q_p = 6.21\ (4290) = 26,700$ cfs by a slide-rule computation. This is the peak rate without structures in place.

2. Determine r.
From Equation 17-42 $r = A^*/A = 103/234 = 0.440$

3. Determine the flood peak for the watershed with structures in place.
Use Equation 17-52 with $(k\ A^h) = 4,290$ cfs/in. from step 1; Q = 6.21 inches, as given; r = 0.440, from step 2; and Qs = 4.5 inches, $q^* = 15$ csm, and $A^* = 103$ square miles as given in Example 17-17. Then $q_p' = 4290(6.21 - 0.440(4.5)) + 15(103) = 18160 + 1540 = 19,700$ cfs.

Note that the effect of the release rate on reducing the storm runoff amount is not taken into account in this example. This means that the peak of 19,700 cfs is slightly too large and that this approach gives a conservatively high answer.

If the storage capacities of the structures vary then Equation 17-52 is used with (Q - r Qs) computed by a more detailed method, as shown in the following example.

Example 17-17.--A watershed of 311 square miles has a system of floodwater retarding structures controlling a total of 187 square miles and having average release rates of 8 csm. Storage capacities of the structures are shown in column 3 of Table 17-22; these are the capacities before emergency spillway discharge begins. When the storm runoff is uniformly 7.5 inches over the watershed, what is the peak rate of flow with (a) no structures in place and (b) structures in place?

1. Determine the flood peak for the watershed with structures not in place.
Use the method of Example 17-14. Enter Figure 17-21 with A = 311 square miles and find q_p/Q = 4,800 cfs/in. This is (k A^h) in Equation 17-52. Multiplying that result by Q = 7.5 inches gives q_p = 36,000 cfs by a slide-rule computation. This is the peak rate without structures in place.

2. Compute the equivalent of (r Qs) in Equation 17-52.
The factor (r Qs) can also be expressed as:

$$(r\ Qs) = \frac{\Sigma(A_x \times Qs_x)}{A} \qquad \text{(Eq. 17-56)}$$

where A_x is the drainage area in square miles of the x-th structure and Qs_x is the reservoir capacity in inches for that structure. In Table 17-22 each drainage area of column 2 is multiplied by the respective storage of column 3 to get the entry for column 4. But note that when the storage exceeds the storm runoff it is the storm runoff amount, in this case 7.5 inches, which is used to get the entry for column 4. Equation 17-56 is solved for (r Qs) by dividing the sum of column 4 by the total watershed area:

$$(r\ Qs) = \frac{967.26}{311} = 3.11 \text{ inches}$$

(Note: Column 4 is not needed if the calculations are made by accumulative multiplication on a desk-calculator.)

3. Determine the flood peak for the watershed with structures in place.
Use Equation 17-52 with (k A^h) = 4,800 cfs/in. from step 1; Q = 7.5 inches, as given; (r Qs) = 3.11 inches as computed in step 2; and q^* = 8 csm and A^* = 187 square miles, as given. This gives: q_p' = 4800(7.5 - 3.11) + 8(187) = 21100 + 1495 = 22,595 cfs, which is rounded to 22,600 cfs. This is the peak rate with structures in place.

DISCUSSION. These examples are a sample of the many ways in which the unit-hydrograph method of routing can be used. Accuracy of the method depends on what has been ignored, such as variable release rate, surcharge storage, and so on. In general, the method gives conservative results--that is, the effects of structures, for example, are usually underestimated so that the peak rate is slightly too high.

The examples also show that as the problem contains more details the procedure gets more complex. It is easily possible to make this "short-cut" method so complicated it becomes difficult to get the solution. For this reason, and for reasons of accuracy, it is better to use the SCS electronic-computer program for complex routing problems.

Table 17-22 Area and storage data for Example 17-17.

Floodwater retarding structure	Contributing drainage area (sq. mi.)	Storage (in.)	A_x x Qs_x (sq. mi. x in.)
(1)	(2)	(3)	(4)
1	14.2	6.1	86.62
2	8.3	6.8	56.44
3	3.7	9.2	21.75*
4	9.4	5.5	51.70
5	17.1	4.5	76.95
6	25.2	3.7	93.24
7	12.9	5.1	65.79
8	6.0	7.5	45.00
9	3.2	10.0	24.00*
10	5.5	8.0	41.25*
11	21.0	4.0	84.00
12	16.4	4.3	70.52
13	9.3	6.5	60.45
14	11.6	5.5	63.80
15	12.5	5.3	66.25
16	10.7	5.0	53.50
		$\Sigma(A_x \text{ x } Qs_x)$ =	967.26

* This is (drainage area) x (storm runoff of 7.5 inches) because the storage greater than the runoff is ineffective and should not be used in the computation.

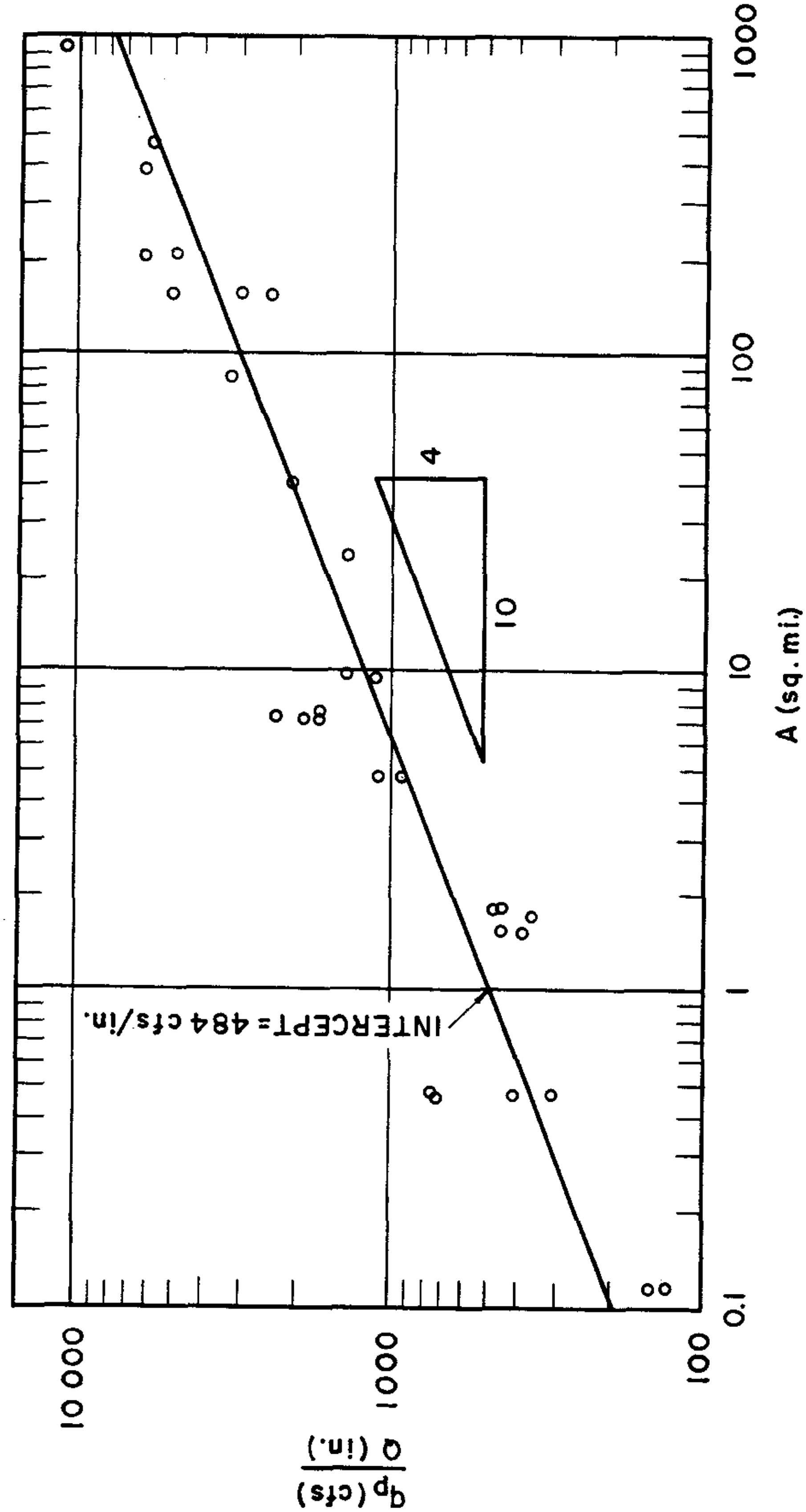

Figure 17-21. q_p/Q versus A for a typical physiographic area.